

# Hill Slope Unsaturated Hydraulic Conductivity: A Comparison of Various Methods

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Keywords	Abstract
Infiltration Tension Disc Infiltrometer Saturated and Unsaturated hydraulic Conductivity	Hydraulic conductivity can be estimated by several methods using tension disc infiltrometer data. In this study three methods were compared together for estimating saturated and unsaturated hydraulic conductivity. Field experiments were conducted in a loamy soil with different slope gradient in Gonbad research station, Hamadan, Iran. Soil surface slope of, 0, 10, 20, 30 and 40 degrees were selected. For each slope, water infiltration experiments were carried out using a tension infiltrometer at water pressure heads of 0, -6, -9 and -15 cm, in three replications. Saturated hydraulic conductivity also estimated using falling head procedure in laboratory. Results indicated that at the same tension the hydraulic conductivity values decreased with increase in slope gradient. In the same slope gradient, the hydraulic conductivity values also decreased with increase in tension values. Ankeny et al. method has better accuracy than white and sully method. Regression method has best closeness in comparison with the other two methods. The correlation of Ankeny et al. procedure as a function of White and Sully, regression procedure as a function of White and Sully procedure and regression procedure as a function of Ankeny et al. procedure were 0.9914, 0.924 and 0.8942 respectively, which are acceptable.

## 1. Introduction

Tension disc infiltrometer can be used for understanding water and solute movement through macropores and the soil matrix near saturation [1; 2; 3]. Soil hydraulic properties include hydraulic conductivity as a function of both soil water pressure and soil water content, and the soil moisture retention relationship [4]. Soil hydraulic conductivity is needed for understanding water balance, irrigation and transport processes. Furthermore, saturated and near-saturated hydraulic conductivity of surface soils influences the partition of irrigation water, rainfall and snowmelt into runoff and soil water storage [5]. Topography or slope gradient, pore-size distribution and pore continuity, and land use are among the main soil and management factors that affect hydraulic properties of surface soils [6; 7]. Several researchers have reported that topography or land slope influences soil properties such as saturated and unsaturated hydraulic conductivity [5; 8; 9; 10]. Therefore, surface soil hydraulic properties may vary between level and sloping landscapes. Some measurement techniques and instruments available for sloping lands include the use of excavated trenches [11], tensiometers, piezometers and lysimeters [12], and hillslope infiltrometer [13]. These methods, however, are time consuming, destructive and tedious to perform under field conditions. Tension disc infiltrometer [14] provide simple, cost effective, non destructive or less destructive and convenient means of in situ measurements of surface soil hydraulic properties [5]. Tension infiltrometers have been widely used for the estimation of soil hydraulic conductivity near saturation [15], sorptivity [16; 17], mobile-immobile water content [18] and water-conducting porosity [1; 19; 20]. Hydraulic conductivity is the proportionality factor in Darcy's law as applied to the viscous flow of water in soil, i.e., the flux of water per unit gradient of hydraulic potential [5]. Therefore, it is the ability of the soil to transmit water (under standard temperature condition) in response to an energy gradient. Unsaturated hydraulic conductivity, on the other hand, is the conductivity at a given water pressure (less than zero) ( $K(h)$ ) or water content (less than saturation) ( $K(\theta)$ ), and thus is a function of soil water content ( $\theta$ ) or water pressure ( $h$ ). Tension disc infiltrometer, which is very popular for measuring surface soil hydraulic properties, was used in this study for unconfined 3-D infiltration measurements. This tool consists of three major components, namely a bubble tower, water reservoir, and a circular disc. Disc is to establish hydraulic continuity with the soil. Infiltration under the tension infiltrometer may be considered as axisymmetric 3-D water flow in a variably saturated porous medium. There are many methods to estimate the ( $K(h)$ ) from tension disc infiltrometer such as White and Sully [21], Smettem and Clothier [22], Ankeny et al. [15] and Logsdon and Jaynes (regression) [3] methods. Most method for estimating  $K(h)$  from tension disc infiltrometer data are based on wooding analysis. For unconfined steady state infiltration under a disc it can be written [23]:

$$q_{ss} = \pi R^2 K(h) + 4R\phi(h) \quad (1)$$

Where  $q$  is the steady state infiltration rate [ $L^3T^{-1}$ ],  $R$  is the radius of disc of infiltrometer [ $L$ ] and  $\phi(h)$  is matric flux potential that defined as:

$$\phi(h) = \int K(h) dh \quad (2)$$

### White and Sully method

[21] suggested an equation for estimation of  $\phi(h)$  as follow:

$$\phi(h) = \frac{bS^2}{\Delta\theta} \quad (3)$$

Where  $b$  is an experimental factor between 0.5 and 0.785,  $\Delta\theta$  is the volumetric water content during infiltration, and the sorptivity,  $S$  can be estimated from early infiltration rate as follow:

$$S = \frac{2qt^{0.5}}{\pi R^2} \quad (4)$$

Where  $q$  is early infiltration rate and  $t$  is time from infiltration start. By substituting Eq.(3) into Eq.(1) it can be written:

$$q_{ss} = \pi R^2 K(h) + 4R \frac{bS^2}{\Delta\theta} \quad (5)$$

Reasonable approximation for  $b=0.55$  has been reported by [22]. Therefore  $K(h)$  can be estimated by Eq.(5). Most problem of White and Sully method include soil must be reasonably dry before infiltration and accurate infiltration rate measurements be made during early time. In addition if  $K(h)$  is to be measured at more than one pressure head, soil must be allowed to dry between measurements [24]. If experiments carried out in different location, resulting in increased variability of soil properties due to spatial variation of soil properties.

### **Smettem and Clothier method**

To reduce the practical difficulties in measuring the early infiltration rate, [22] used infiltrometers with two or more disc diameter. For two infiltrometer disc Eq. (5) can be written and result two equations with two unknowns including  $K$  and  $S$ , that can be solved easily. In this method [22] recommended that  $R_1 > R_2$ . Smettem and Clothier method also has some problem such as variability of soil properties at different points of experiments.

### **Ankeny et al. method**

[15] started with Eq. (1) and measured infiltration rates at two wetting pressure heads that gives four unknowns (include  $K(h_1)$ ,  $K(h_2)$ ,  $\varphi(h_1)$  and  $\varphi(h_2)$ ) with two equations as follow:

$$q(h_1) = \pi R^2 K(h_1) + 4R\varphi(h_1) \quad (6)$$

$$q(h_2) = \pi R^2 K(h_2) + 4R\varphi(h_2) \quad (7)$$

[15] assumed a constant ratio between  $K$  and  $\varphi$ , and an approximate expression for the difference  $\varphi(h_1) - \varphi(h_2)$ . Then they obtained three equations with three unknowns, which could be solved for two hydraulic conductivities ( $K(h_1)$ ,  $K(h_2)$ ) as follow:

$$K(h_1) = \frac{q(h_1)}{\pi R^2 + 2R(h_1 - h_2) \left\{ 1 + \frac{q(h_2)}{q(h_1)} \right\} / \left\{ 1 - \frac{q(h_2)}{q(h_1)} \right\}} \quad (8)$$

$$K(h_2) = \frac{q(h_2)}{q(h_1)} K(h_1) \quad (9)$$

In Ankeny et al. method measurements conducted at one place and different pressure heads, therefore the spatial variability effect of soil properties reduced.

### **Logsdon and Jaynes (regression) method**

[3] started with Eq. (1) and substituting the expression for the exponential relationship between  $K(h)$  and  $h$  [25] into wooding equation, derived the following equation:

$$q_{ss}(h)/(\pi R^2) = K(0) \exp(\alpha h) + [4K(0) \exp(\alpha h)]/(\pi R\alpha) \quad (10)$$

Which  $\alpha$  is a constant and  $K(0)$  is the saturated hydraulic conductivity. Eq. (10) has two unknown ( $\alpha$  and  $K(0)$ ). With infiltrometer measurements made at two or more wetting pressure heads, a nonlinear least squares regression of Eq. (10) can be used to estimate the two unknowns.

In this study field experiments carried out at 4 different pressure heads, 5 different slope gradients and 3 replications in Gonbad research station of Hamadan province, Iran, that include homogenous and isotropic soil, then  $K(h)$  values at different pressure heads estimated using four mentioned methods and compared together.

## **2. Materials and Methods**

### **2.1. Study area**

Gonbad research station, Hamadan, Iran (48° 42.14' N lat., 34° 41.74' W long. and 2170 m elevation from sea level.) selected for this study. Total area of this station is 740 ha and its average annual rainfall and land slope are 560 mm and 28.8 degree, respectively. It generally is a high slope basin. A soil profile with 1.5 m length, 1.5 m width and 2 m depth was excavated. Soil layer was homogenous and no abrupt changes in soil texture and soil layer were observed within 2 m of the soil profile.

### **2.2. Slope treatments**

Five various soil surface slope gradients including 0- , 10- , 20- , 30- , and 40- degree slopes were selected in the area. For each slope gradient, water infiltration experiments were carried out by a tension infiltrometer at tensions 0, 6, 9 and 15 cm of water in three replications. Totally 60 water infiltration experiments were carried out in five different surface slopes, four tensions and three replications ( $5 \times 4 \times 3 = 60$ ).

### **2.3. Field and laboratory Experiments**

To determine the soil physical and chemical properties in different slopes, for each slope three disturbed and three undisturbed samples (0.05 m in diameter and 0.05 m in height) were taken from areas next to the measurement sites. Bulk density, particle density and porosity were estimated using Flint and Flint (2002) method. Sand, silt and clay percentage were also estimated using the hydrometer method. Based on this percentage the texture of the soil was determined by using a textural triangle. Soil in the experimental site is loamy texture (Table 1).

To estimate saturated and unsaturated hydraulic conductivity, a tension infiltrometer with a 0.2 m diameter disk (soil measurement systems, Tuscon. Az) was used. At first the location of experiment was selected and then a thin layer ( $5 \times 10^{-3}$  m) of moist fine sand was applied over the

prepared surface at each measurement location in a circular area with a diameter equal to the diameter of infiltrometer disk. The hydraulic conductivity of testing sand must be more than that of the experimental soil. Applying the fine sand has two advantages as follow[5]:

(1) The sand prevents tearing the nylon mesh attached to the infiltrometer disk and (2) This smoothes out any irregularities of the soil surface and ensures good contact between the soil surface and the infiltrometer membrane.

After preparation of the experiment location, tension infiltrometer instrument was regulated in given tension and was placed on it. The amount of infiltration into the soil was measured by recording the water level falling in the graded reservoir tower as a function of time. When the amount of water entered into the soil did not change with time for three consecutive measurements taken at 5-minute intervals, steady state flow was assumed and the corresponding infiltration rate was calculated based on the last three measurements. Generally, steady state flow was achieved within 30 to 60 min for the tension infiltrometer.

Furthermore, saturated hydraulic conductivity measured using falling head method in the laboratory and results of this method compared with three White and Sully, Ankeny et al and regression methods results. Smettem and Clothier method didn't use in this study because one diameter of disc (20 cm) was used in this study.

### 3. Finding and Discussion

For each slope, three disturbed and three undisturbed samples were taken from areas next to the measurement sites. Some soil physical and chemical properties were determined in laboratory and illustrated in table 1. According to table 1 soil of study area is almost homogenous. Hydraulic conductivity values were calculated using tension infiltrometer data by three White and Sully, Ankeny et al and regression methods.

#### 3.1. Falling head method

Undisturbed samples used to determination of saturated hydraulic conductivity by falling head method in the laboratory. Falling head method overestimated the hydraulic conductivity values than other methods. Table 2 showed the saturated hydraulic conductivity values for different slope gradient using falling head method. With increase in slope gradient, saturated hydraulic conductivity was decreased.

#### 3.2. White and Sully method

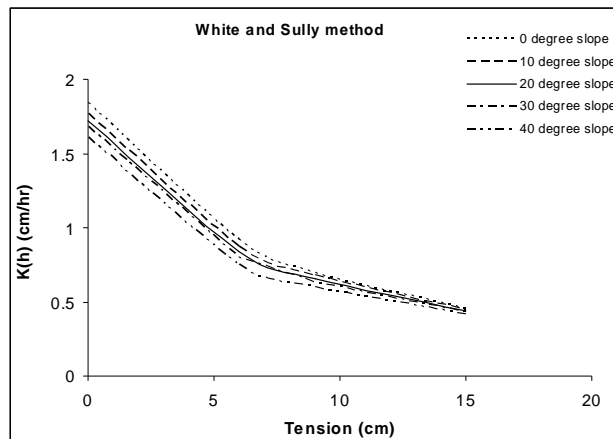
For each tension, by calculating the soil sorptivity in the early time of each infiltration experiment, saturated ( $h=0$ ) and unsaturated ( $h=6, 9, 15$  cm) hydraulic conductivity values calculated by White and Sully method (Eq. (5)). In this study  $b=0.55$  has been assumed according to Smettem and Clothier approaches. Fig. 1 illustrates the hydraulic conductivity values for different tensions and slope gradients calculated by White and Sully procedure. To convert three dimensional infiltration measurement to  $K(h)$ , White and Sully method is suitable, but required determinations of  $S$  and initial and final soil water content.

**Table 1.** Some selected soil physical and chemical properties of the experimental site

Parameter	Slope gradient (degree)				
	0	10	20	30	40
Bulk density ( $\text{gr}/\text{cm}^3$ )	1.66	1.67	1.68	1.68	1.69
Particle density ( $\text{gr}/\text{cm}^3$ )	2.58	2.57	2.57	2.57	2.58
Sand (%)	39.3	38.4	38.9	38.1	40.2
Silt (%)	38.1	37.2	37.6	39.2	38.5
Clay (%)	22.6	24.4	23.5	22.7	21.3
Porosity (%)	35.66	35.02	34.63	34.63	34.5
O. M. content (%)	0.6	0.3	0.6	0.5	0.6
PH	7.9	7.9	7.8	7.9	7.8
SAR( $\text{mmol}/\text{lit}$ ) <sup>0.5</sup>	28.03	28.21	27.95	28.12	28.15
$\text{EC}_e(\text{ds}/\text{m})$	0.46	0.45	0.45	0.47	0.45

**Table 2.** Saturated hydraulic conductivity values for different slope gradient calculated by falling head method

Slope gradient (degree)	0	10	20	30	40
$K_s$ (cm/hr)	2.21	2.12	2.02	1.94	1.86



**Figure 1.** Trend of  $K(h)$  changes with  $h$  increasing calculated using White and Sully method in different slope gradients

This method underestimated the saturated hydraulic conductivity respect to falling head method. This method underestimated also the saturated and unsaturated hydraulic conductivity respect to Ankeny et al and regression methods.

### 3.3. Ankeny et al method

By writing Wooding's equation for each pair of tensions two equations with two unknown can be obtained and solved. To estimating saturated and unsaturated hydraulic conductivity values by Ankeny et al method equations (8) and (9) were used. Saturated and unsaturated hydraulic conductivity values calculated by Ankeny et al method have been illustrated in Fig. 2. This method overestimated the saturated hydraulic conductivity respect to falling head method and overestimated the saturated and unsaturated hydraulic conductivity respect to White and Sully and regression methods.

### 3.4. Logsdon and Jaynes(Regression) method

By substituting [25] exponential unsaturated hydraulic conductivity equation into Wooding's equation a nonlinear least squares regression of Eq. 10 used to estimate the two unknown ( $\alpha$  and  $K(0)$ ). Table 3 illustrated the  $\alpha$  and  $K(0)$  for various slope gradients which calculated using optimizing and nonlinear least squares regression procedure for four tensions (0, 6, 9 and 15 cm).  $\alpha$  values decrease with increase in slope gradient. Saturated hydraulic conductivity ( $K(0)$ ) also decreased with increase in slope gradient. Fig.3 showed the trend of hydraulic conductivity changes with tension changes in different slope gradient calculated by regression method.

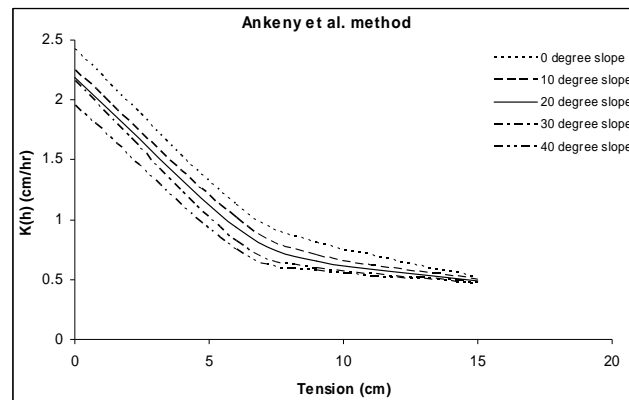


Figure 2. Trend of  $K(h)$  changes with  $h$  increasing calculated using Ankeny et al method in different slope gradients

Table 3. Constant parameter ( $\alpha$ ) and saturated hydraulic conductivity ( $K(0)$ ) in different slope gradient for regression procedure

Parameter	Slope gradient (degree)				
	0	10	20	30	40
$\alpha$ (1/cm)	0.0892	0.0863	0.0853	0.0852	0.0845
$K(0)$ (cm/hr)	2.25	2.09	2	1.92	1.87

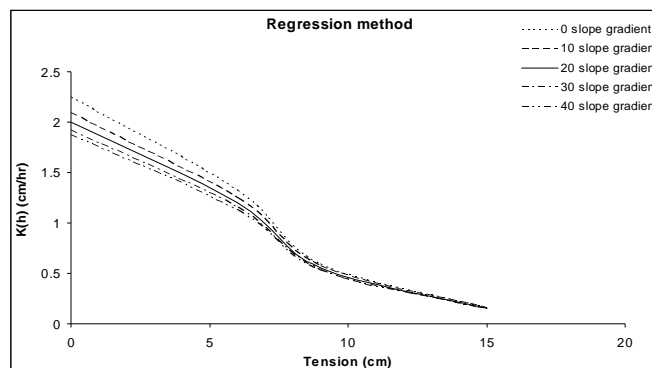


Figure 3. Trend of  $K(h)$  changes with  $h$  increasing calculated using regression method in different slope gradients

### 3.5. Saturated and unsaturated hydraulic conductivity

In estimation of saturated hydraulic conductivity, regression results have better closeness than Ankeny et al. and White and Sully procedures respect to falling head procedure. Figs. 4 to 6 illustrate the comparison of results of three methods (White and Sully, Ankeny et al. and regression) respect to falling head method in estimation of saturated hydraulic conductivity. For slope gradients of 0, 10, 20, 30 and 40 degree of slope, corresponding correlation coefficient were 0.0.899, 0.842, 0.812, 0.769 and 0.743 (for  $n=12$  data) respectively. Figs. 7 to 9 also illustrate the comparison of results of three methods (White and Sully, Ankeny et al. and regression) in estimation of saturated and unsaturated hydraulic conductivity.

### 3.6. Comparison of Saturated Hydraulic Conductivity

To compare results of various methods in estimating saturated hydraulic conductivity, results of each method compared with results of falling head method in different slope gradients. According to Figs.4 to 6, White and Sully method underestimated and Ankeny et al. method overestimated the saturated hydraulic conductivity at all slope gradient. Regression method has best closeness with falling head results. The slope of the regression equation for White and Sully, Ankeny et al. and Logsdon and Jaynes procedures as a function of falling head procedure were 0.6237, 1.1637 and 1.0619 respectively. The slope of the White and Sully procedures as a function of falling head procedure only was significantly different from one. The correlation of White and Sully, Ankeny et al. and Logsdon and Jaynes procedures as a function of falling head procedure were 0.9891, 0.9144 and 0.9666 respectively, which are acceptable.

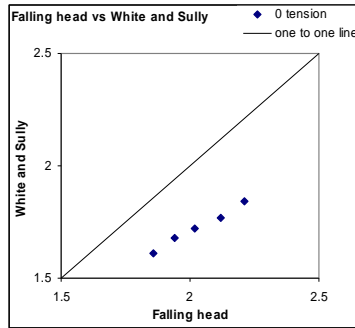


Figure 4. Comparison of saturated hydraulic conductivity calculated by two various methods

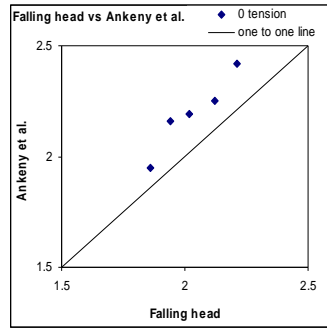


Figure 5. Comparison of saturated hydraulic conductivity calculated by two various methods

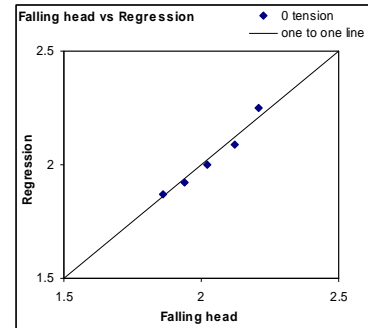


Figure 6. Comparison of saturated hydraulic conductivity calculated by two various methods

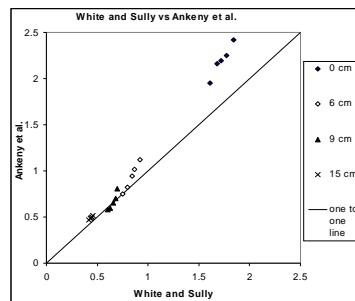


Figure 7. Comparison of unsaturated hydraulic conductivity calculated by two various methods

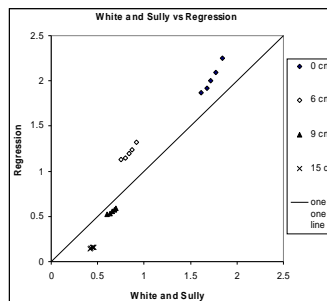


Figure 8. Comparison of unsaturated hydraulic conductivity calculated by two various methods

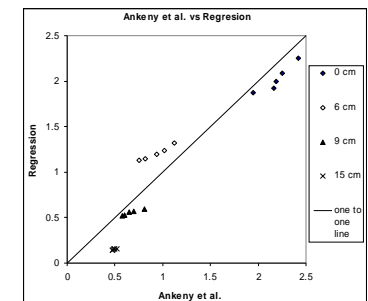


Figure 9. Comparison of unsaturated hydraulic conductivity calculated by two various methods

## 4. Conclusion

According to Figs. 7 to 9 unsaturated hydraulic conductivity for Ankeny et al. procedure as a function of White and Sully procedure and Logsdon and Jaynes procedure as a function of White and Sully have poor closeness together. Unsaturated hydraulic conductivity for regression procedure as a function of Ankeny et al. procedure has an acceptable closeness together. The slope of the regression equation for Ankeny et al. procedure as a function of White and Sully procedure, regression procedure as a function of White and Sully and regression procedure as a function of Ankeny et al. procedure were 1.3695, 1.3962 and 0.9986 respectively. The slope of the regression equation for Ankeny et al. procedure as a function of White and Sully procedure and regression procedure as a function of White and Sully are significantly more than one to one slope, but the slope of the regression equation for regression procedure as a function of Ankeny et al. procedure is about one to one slope. Ankeny et al. method overestimated the saturated and unsaturated hydraulic conductivity respect to White and Sully method at all tensions. White and Sully method overestimated the hydraulic conductivity respect to regression method at high tensions ( $h=9, 15$  cm) and underestimated at low tension ( $h=0, 6$  cm). Ankeny et al. method also overestimated the hydraulic conductivity respect to regression method at high tensions ( $h=9, 15$  cm) and underestimated at low tension ( $h=0, 6$  cm). The correlation of Ankeny et al. procedure as a function of White and Sully, regression procedure as a function of White and Sully procedure and regression procedure as a function of Ankeny et al. procedure were 0.9914, 0.924 and 0.8942 respectively, which are acceptable (for  $n=20$  data).

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