

An integrated fuzzy–fractal geochemical index for gold exploration targeting in the Saqqez mineralization belt, Iran

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Received: 5 January 2026 / Accepted: 20 October 2026

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Abstract

In complex geochemical environments, separating anomalies from background values remains a major challenge. This study proposes an innovative integrative framework combining concentration-number (C–N) fractal modeling and fuzzy c-means (FCM) clustering to improve gold geochemical anomaly mapping in the Saqqez mineralization area, Sanandaj–Sirjan Zone. While fractal modeling is used for data-driven anomaly identification and background suppression, FCM clustering identifies multielement patterns related to mineralization. The main novelty of this research is the development of the Integrated Fractal–FCM Anomaly Index (IFFAI). This continuous, dimensionless index integrates standardized fractal-based anomaly intensity and FCM membership degrees, enabling the simultaneous quantification of anomaly strength and classification confidence. The results demonstrate that while standalone fractal modeling and FCM clustering fail to fully delineate all mineralized zones, the integrated IFFAI approach successfully identifies all known gold mineralization zones in the Saqqez region with a coherent and continuous pattern. Furthermore, by combining fractal anomaly classes with FCM-derived mineralization affinity factors, geochemical samples were systematically prioritized, showing strong spatial correlation with active exploration targets. Compared to standalone methods, the proposed framework provides significant improvements in spatial resolution, anomaly continuity, and background noise reduction. Overall, this method serves as an effective decision-support tool for geochemical anomaly mapping and exploration target prioritization.

Keywords Geochemical anomaly mapping · Fuzzy logic · Fractal modeling · Geochemical sample prioritizing · Saqqez area

Introduction

Several theses have been written regarding gold mineralization in the Saqqez region, which have demonstrated a strong structural control on gold mineralization. Heydari¹ showed that gold mineralization at Kervian, southwest of Saqqez, is closely associated with ductile shear zones, intense deformation, mylonitization, and hydrothermal alteration. At the Qolqoleh deposit, Aliyari² demonstrated that gold mineralization is mainly hosted by

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ductile–brittle shear zones and that higher gold grades are associated with strongly deformed, silicified mylonitic and ultramylonitic domains as well as quartz–sulfide veins. Subsequent studies confirmed that brittle deformation provided favorable sites for the emplacement of auriferous fluids and the development of Au-bearing quartz–sulfide veins. Nosratpoor³ specifically investigated gold mineralization within the Ghabaghlojeh shear zone, further emphasizing the importance of shear structures in controlling gold mineralization in southwestern Saqqez. Seyedrahimi-Niaraq⁴ investigated the geochemical characteristics of shear-zone-hosted gold mineralization in southwestern Saqqez using U-spatial statistics combined with probability-plot modeling. Their results identified three mineralized areas and indicated that the Au anomalies were related to two stages of hydrothermal mineralizing events with different intensities. The second hydrothermal event was more intense and produced the most pronounced Au anomalies. These studies collectively indicate that deformation, shear-zone development, hydrothermal alteration, and structural permeability played major roles in the localization of gold mineralization in the Saqqez gold district. In this study, we proposed a novel integrated fuzzy–fractal index for geochemical targeting and sample prioritization in the Saqqez mineralization area.

Geochemical anomaly mapping constitutes a fundamental pillar of mineral exploration, as it enables the reliable delineation of mineralization-related halos from natural background variations. Accurate anomaly identification facilitates more focused exploration strategies, reduces unnecessary sampling costs, and significantly improves the success rate of subsequent exploration stages^{5,6}. However, geochemical datasets commonly exhibit intrinsic complexities such as multipopulation distributions, strong spatial heterogeneity, censored values, and elevated noise levels. These characteristics pose substantial challenges to conventional anomaly detection methods, particularly when both the intensity of anomalies and the confidence associated with their identification must be considered simultaneously^{7,8}.

Classical statistical approaches, including mean- or quantile-based thresholds, standard deviation, and median absolute deviation (MAD), are widely applied due to their simplicity but often fail in the presence of skewed distributions, outliers, and overlapping geochemical populations⁸. Traditional statistical methods frequently struggle to separate these populations due to their reliance on assumptions of normal or log-normal distributions. To address these issues, fractal-based approaches offer significant advantages. Unlike classical statistics, fractal analysis does not require any prior distribution assumptions and is capable of modeling spatial heterogeneity, non-linear geological features, and scale-invariant behaviors. These methods allow for a more realistic characterization of complex geochemical systems by identifying distinct populations based on their scaling properties^{9–11}. The Fractal-based methods such as concentration–area (C–A) and concentration–number (C–N) models have been introduced to exploit the scale-invariant nature of geochemical data and to derive data-driven thresholds without relying on distributional assumptions^{7,12}.

Over the past few decades, fractal and multifractal models have become established mathematical frameworks in mineral exploration for delineating geochemical anomalies, defining threshold values, and identifying mineralization stages. The spatial distribution of gold grades and associated pathfinder elements typically exhibits strong non-linear behavior due to complex, continuous geological processes and structural controls. Consequently, fractal analysis has been widely applied as a spatial proxy to map these complex distributions. For instance, fractal dimension gradients have been effectively utilized as a reliable spatial proxy to interpret the migration pathways of auriferous fluids and constrain economic mineralized zones, particularly in Carlin-type gold deposits¹³. In these systems, spatial variations in fractal dimension values reflect the mechanisms of dispersion, aggregation, and structural control during the evolution of the mineralizing system¹³. Furthermore, regional geochemical exploration has successfully utilized the multifractal model combined with multivariate statistics to filter background noise and isolate localized anomalies associated with gold mineralization in complex metallogenic belts^{14,15}.

In Iranian gold belts, these fractal models have been extensively demonstrated. These applications include discriminating distinct gold mineralization stages, correlating geochemical populations with field-scale structural and lithological datasets, and mapping vertical grade distributions in major deposits such as the Zarshuran

Carlin-like deposit^{16,17} and the Arabshah epithermal gold system¹⁸. Together, these studies confirm that fractal parameters are not only mathematical constructs but also valid spatial proxies that reflect geological processes and ore-controlling structures, justifying their integration into multi-criteria targeting frameworks such as the index proposed in this study.

Despite their effectiveness in objectively separating background and anomalous populations, fractal and probability plot (PP) approaches are largely univariate and therefore neglect inter-element correlations and paragenetic relationships, which may lead to incomplete characterization or overestimation of geochemical anomalies¹⁹.

To address these issues, multivariate and intelligent techniques have been increasingly adopted in geochemical anomaly mapping. Principal component analysis (PCA) is commonly used to extract latent structures and reduce dimensionality; however, it is often insufficient for resolving overlapping populations when applied alone^{20,21}. More advanced machine-learning and clustering algorithms including fuzzy c-means (FCM), artificial neural networks (ANN), support vector machines (SVM), and random forests (RF) have demonstrated strong abilities to detect nonlinear patterns and to handle uncertainty inherent in complex geochemical datasets^{22,23}. Recent studies indicate that integrating classical geochemical techniques, such as fractal modeling, with intelligent multivariate approaches can significantly enhance anomaly discrimination and improve the signal-to-noise ratio in geochemical maps^{24,25}. Among these methods, the FCM algorithm has gained particular attention due to its ability to explicitly represent uncertainty by assigning fuzzy membership degrees to each sample rather than enforcing crisp class boundaries²². This characteristic makes FCM especially suitable for multipopulation and geologically complex environments where mineralized, altered, and background samples commonly overlap. Numerous studies have shown that FCM-derived membership maps exhibit strong spatial correspondence with known mineralization zones and outperform deterministic clustering techniques in delineating geochemical anomalies^{26–28}.

Supervised machine-learning methods, including ANN, SVM, and RF have been widely applied to mineral prospectivity mapping and geochemical anomaly detection. These methods can achieve high predictive performance when sufficient, representative, and reliable labelled data are available. However, their application may be constrained in early-stage exploration, where confirmed mineralized and non-mineralized training samples are limited, unevenly distributed, or affected by uncertainty. Moreover, geochemical anomalies commonly exhibit gradual transitions from background to weakly and strongly mineralized populations rather than sharp class boundaries. Under such conditions, fuzzy clustering provides a suitable alternative because it is unsupervised and does not require predefined class labels. In particular, FCM assigns each sample a degree of membership to several clusters, allowing transitional and uncertain geochemical signatures to be represented explicitly. This probabilistic-like membership information is readily interpretable and can be integrated with fractal characteristics to distinguish background, transitional, and anomalous populations and to prioritize samples based on their relative exploration potential. Therefore, the advantage of fuzzy clustering in the present study is not universal superiority over ANN, SVM, or RF, but its suitability for label-limited geochemical datasets with gradual and uncertain anomaly boundaries.

Although both fractal-based models and fuzzy clustering techniques have proven effective individually, their integration offers a more comprehensive framework for geochemical anomaly analysis. In such a hybrid approach, fractal modeling objectively characterizes anomaly intensity through data-driven thresholding, while FCM captures multivariate associations and quantifies the uncertainty of sample attribution. Unlike conventional studies that merely juxtapose or visually overlay the outputs of different methods, a fully integrated framework enables the transformation of these complementary perspectives into a single, interpretable metric that simultaneously reflects anomaly strength and confidence^{24,25}. Motivated by these considerations, this study proposes a hybrid fuzzy–fractal anomaly index designed to address the intrinsic limitations of standalone methods, particularly in complex geochemical settings such as the Saqqez gold mineralization area. The proposed framework integrates fractal-derived information on gold (Au) anomaly intensity with FCM-based membership degrees associated with mineralization-related clusters, producing a unified, continuous index capable of supporting both anomaly delineation and sample prioritization.

Accordingly, the present research proposes a hybrid fuzzy–fractal framework, termed the Integrated Fractal–FCM Anomaly Index (IFFAI), to improve gold geochemical anomaly mapping in the Saqqez shear-zone mineralization area. The method integrates C–N fractal analysis for objective anomaly thresholding with FCM clustering for capturing multivariate geochemical relationships and sample-membership uncertainty. This integration yields a continuous index that is used to delineate anomalous zones and prioritize samples for exploration. The proposed index is evaluated against standalone fractal and FCM results, together with conventional statistical and multivariate approaches, to demonstrate its ability to enhance anomaly discrimination, reduce background noise, and improve spatial correspondence with known mineralized zones.

The resulting index of IFFAI provides a quantitative measure that bridges the gap between threshold-based anomaly detection and uncertainty-aware clustering. The performance of IFFAI is evaluated through comparison with conventional statistical, fractal, and multivariate approaches applied to the Saqqez dataset, where previous methods such as PP plots, fractal thresholding, and standalone PCA have shown limited effectiveness. The results demonstrate that IFFAI enhances anomaly discrimination, improves boundary definition, and reduces geochemical noise, thereby offering a robust decision-support tool for targeted sampling and exploration planning. Owing to its generic structure, the proposed framework can be readily extended to other geochemical elements and exploration regions.

Material and methods

Geological setting and mineralization

The southwestern part of the Saqqez geological sheet (1:100,000 scale) is situated at the northwestern margin of the Sanandaj–Sirjan metamorphic belt, close to its tectonic junction with the Khoy–Mahabad and Alborz–Azarbaijan structural zones (Fig. 1). This tectonically complex region is characterized by pronounced lithological diversity, reflecting a long and multi-stage geological evolution. The oldest exposed units consist of Precambrian metamorphic rocks, including gneiss, schist, marble, and amphibolite, which form the regional basement. These basement rocks are unconformably overlain by younger sedimentary and igneous successions, comprising clastic–carbonate assemblages, thick Permian limestone sequences, and widespread intrusive bodies of Mesozoic to Cenozoic age. The Kahar Formation, representing rocks of relatively low-grade metamorphism, occurs as an intermediate unit between the high-grade Precambrian basement and younger cover sequences. Metamorphic conditions throughout the area vary considerably, ranging from greenschist to amphibolite facies, indicative of heterogeneous metamorphic overprinting^{19,29}.

Multiple phases of magmatic activity constitute a major geological feature of the study area. The earliest igneous events are represented by Precambrian granitoid intrusions, whereas younger granitoid stocks of Mesozoic and Cenozoic age intrude both the metamorphic basement and Paleozoic sedimentary units. These intrusive bodies are genetically associated with hydrothermal systems that played a significant role in the development of skarn-type metallic mineralization. Subsequent geological processes include the emplacement of Jurassic volcanic and sedimentary successions, consisting mainly of basaltic flows, shale, and sandstone, which locally blanket the older formations. Regional tectonism has profoundly modified the entire lithological assemblage, giving rise to extensive fault networks dominantly oriented in NW–SE and NE–SW directions. Intense deformation within metavolcanic units led to the formation of mylonitic and ultra-mylonitic zones, which constitute the principal hosts for gold mineralization^{19,31}.

Hydrothermal fluid circulation associated with magmatic and tectonic activity extensively altered the host rocks, resulting in the development of secondary mineral assemblages including chlorite, carbonate, sericite, and sulfide minerals, together with pervasive silicification²⁹. Distinct alteration styles such as sulfidization, carbonatization, silicification, sericitization, and chloritization are spatially organized and serve as effective indicators for delineating mineralized zones and assessing proximity to ore bodies¹. Gold mineralization in the area is

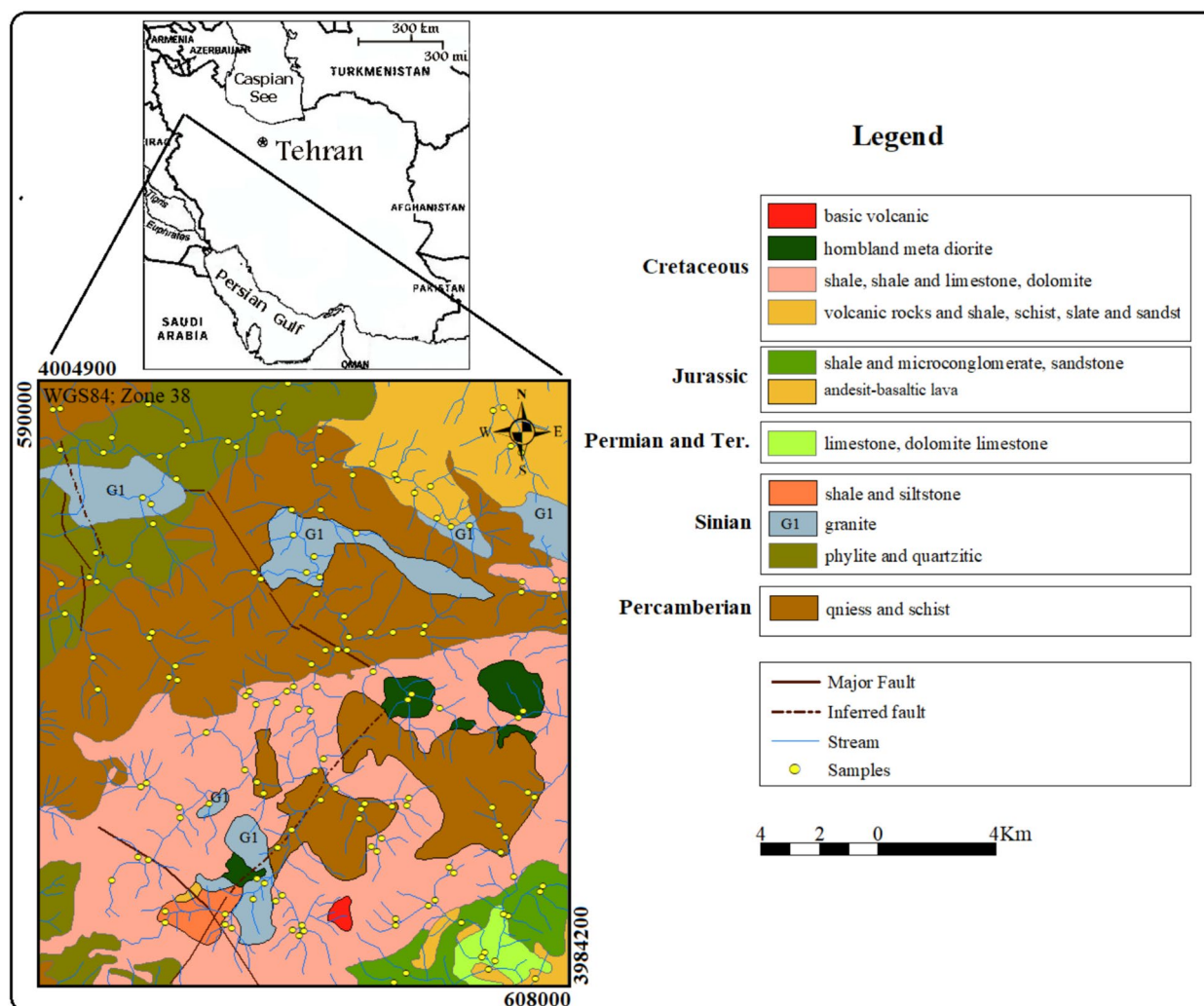


Fig. 1 Geographical location and Geological map of SW-Saqez area (after GSI³⁰).

classified as shear-zone-hosted, occurring predominantly within deformed volcanic and sedimentary lithologies. This mineralization style is typical of orogenic gold systems developed within metamorphic belts under compressional tectonic regimes^{19,32}. Chemical interaction between hydrothermal fluids and granitoid intrusions with the surrounding country rocks facilitated extensive alteration and ore deposition. The resulting alteration halos exhibit a systematic zoning pattern, wherein sericite- and chlorite-dominated assemblages characterize distal zones, whereas more intense alteration and sulfide enrichment are concentrated in proximal zones closely associated with gold-bearing structures³³. Several gold deposits and mineral occurrences identified through previous geological investigations in this area. Ore formation is primarily attributed to metamorphic devolatilization and dehydration processes. The deposits are predominantly hosted by sericite schist, chlorite schist, and mafic to felsic metavolcanic rocks ranging in age from the Devonian to the Triassic. Structurally, mineralization is controlled by ductile to brittle shear zones (Fig. 2a, b), which developed in association with the major deep-seated thrust and normal faults along the southwest of Saqez lineament³⁴. These findings were incorporated into this study and served as reference points for validating the analytical results.

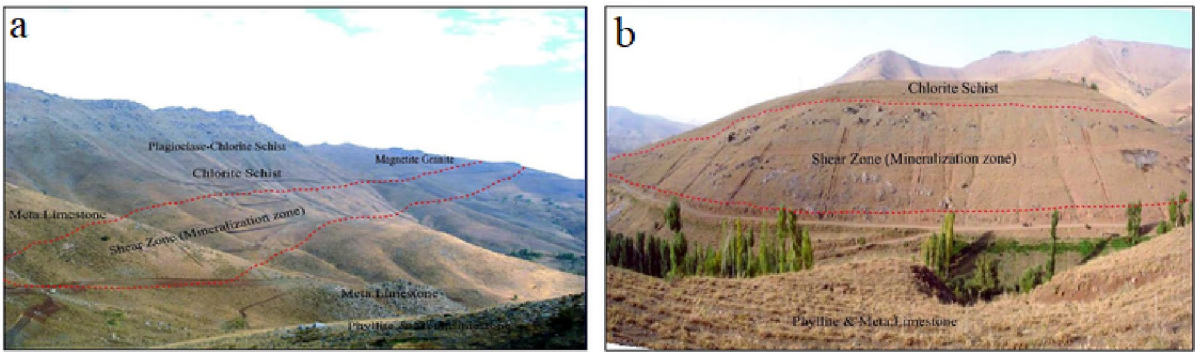


Fig. 2 Field views showing: **(a)** the lithological units and shear zone at the Qolqoleh gold deposit; and **(b)** the lithological units and shear zone at the Ghabaghlojeh gold deposit³⁴.

Table 1 Descriptive statistics of the geochemical variables.

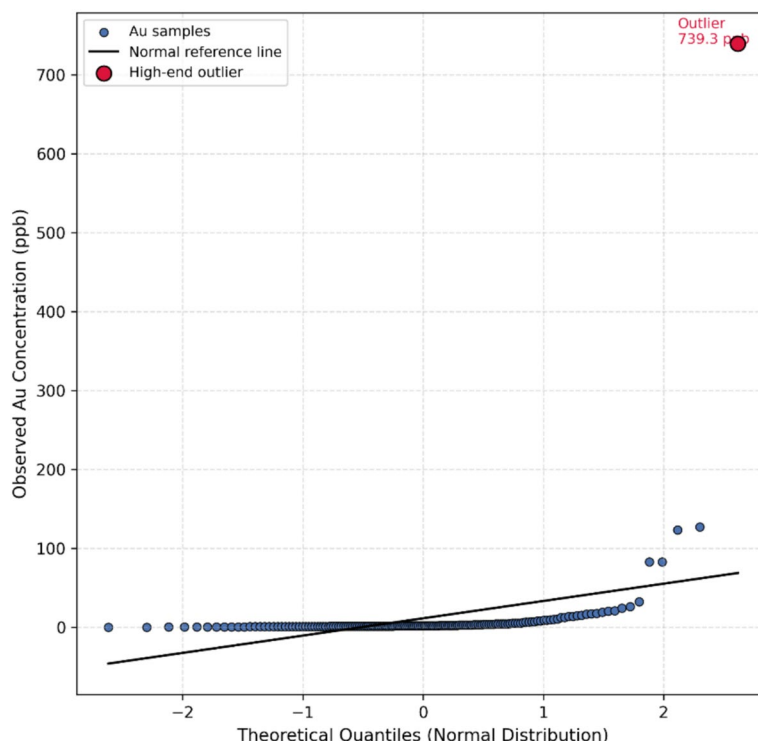
Variable	n	Min	Max	Mean	Median	SD	CV (%)	Skewness	Shapiro–Wilk p
Au	156	0.08	739.3	11.51	2.08	61.09	530	6.34	<0.0001
Ag	156	0.04	0.27	0.09	0.08	0.04	42.59	1.93	<0.0001
As	156	3.06	274.10	47.59	33.14	49.76	104.58	2.15	<0.0001
B	156	6.40	195.70	52.66	49.50	29.83	56.65	0.86	<0.0001
Ba	156	338.00	895.00	459.16	461.50	64.19	13.98	3.58	<0.0001
Be	156	1.00	3.90	2.39	2.40	0.45	18.90	0.26	0.1569
Bi	156	0.10	1.33	0.35	0.33	0.20	58.97	1.71	<0.0001
Co	156	14.60	58.50	28.57	27.65	8.20	28.71	0.74	0.0004
Cr	156	69.40	287.90	149.25	146.25	46.86	31.40	0.68	0.0003
Cu	156	14.40	62.10	28.83	28.00	8.31	28.84	0.63	0.0008
Hg	156	0.02	9.95	0.19	0.05	0.91	472.79	9.20	<0.0001
Mn	156	473.00	2167.00	932.37	856.00	321.36	34.47	1.13	<0.0001
Mo	156	0.42	6.31	1.36	1.08	0.95	69.40	2.32	<0.0001
Ni	156	23.10	209.20	82.34	71.95	35.94	43.65	1.06	<0.0001
Pb	156	12.60	68.40	30.26	28.65	11.42	37.74	0.98	<0.0001
Sb	156	0.51	160.00	9.61	2.41	22.85	237.73	4.34	<0.0001
Sn	156	1.60	10.50	4.37	4.00	1.67	38.19	1.31	<0.0001
Ti	156	3007.00	9823.00	5170.12	5010.50	1210.58	23.41	0.74	0.0009
V	156	66.30	273.80	152.03	153.55	46.12	30.33	0.22	0.0177
W	156	2.30	11.60	4.06	4.00	1.55	38.20	1.86	<0.0001
Zn	156	49.00	211.00	95.59	93.00	30.18	31.57	0.63	<0.0001

Dataset description

In this study, a geochemical dataset derived from surface sampling of the study area was used, comprising analytical results for 21 elements. The descriptive statistics of the analyzed geochemical dataset indicate a strongly heterogeneous distribution of element concentrations, with most variables exhibiting positive skewness and non-normal behavior according to the Shapiro–Wilk test. Gold shows the most pronounced departure from normality, with an extreme right-skewed distribution driven by a small number of anomalously high values. Such statistical characteristics are typical of exploration geochemical data and justify the use of robust anomaly-detection methods rather than classical parametric thresholds. The revised Table 1 summarizes the full descriptive statistics of the 21 analyzed elements, including measures of central tendency, dispersion, skewness, and normality assessment.

To examine the statistical behavior of the gold data, a Q–Q plot was constructed for Au concentrations (Fig. 3). The Q–Q plot (clearly exhibits a strong right-skewed pattern, which is highly characteristic of lognormal or

Fig. 3 The Q–Q plot of Au concentrations (ppb) with geochemically significant outlier.



exponential distributions commonly observed in gold geochemical datasets. This deviation from normality highlights the highly skewed nature of the data and the presence of high-end outlier values. The tail of the distribution, clearly deviating from the reference line. The value of this isolated outlier is 739.3 ppb, which was not interpreted as a confirmed geological anomaly, but rather as a suspicious analytical outlier likely reflecting a measurement-related artifact or a localized nugget effect. To mitigate the disproportionate influence of this non-representative value on continuous spatial and fractal modeling, it was Winsorized using the mean plus three standard deviations criterion and capped at 194 ppb. The resulting corrected dataset was subsequently employed for further analysis and modeling.

Concentration-number (C–N) fractal analysis

The concentration–number (C–N) fractal method is a scale-invariant technique widely used to separate geochemical populations by examining the relationship between element concentrations and their cumulative frequency distribution¹². This approach relies on the assumption that geochemical datasets often exhibit fractal or multifractal behavior, reflecting the superposition of multiple geological processes operating at different scales. As a result, the method is particularly suitable for complex, heterogeneous, and multimodal datasets, including those affected by censored measurements. To implement the C–N fractal analysis, concentration values of the target variable are first sorted in ascending order. For each concentration level C , the cumulative number of samples with concentrations greater than or equal to C is calculated. The relationship between concentration and cumulative sample number follows a power-law function expressed as:

$$N(C) = k \cdot C^{-D} \quad (1)$$

where $N(C)$ denotes the cumulative number of samples exceeding concentration C , k is a proportionality constant, and D represents the fractal dimension^{35,36}. When plotted on logarithmic axes, this relationship typically forms multiple linear segments characterized by distinct slopes. Breakpoints between these linear segments correspond

to transitions between different geochemical populations and are interpreted as objective threshold values. Such thresholds are commonly associated with changes in the dominant processes controlling element distribution. In practical applications, these breakpoints can be identified by examining slope variations, applying piecewise linear regression, or using logarithmic derivative analyses^{37–39}.

Segments with lower slope magnitudes generally represent background or low-variability populations, whereas segments with steeper slopes are indicative of anomalous populations characterized by higher variability and enrichment. An advantage of the C–N fractal method is its ability to accommodate left-censored datasets by utilizing the inherent scaling properties of the data, rather than excluding values below analytical detection limits, thereby preserving essential information for threshold determination³⁶. The C–N fractal method provides a data-driven, distribution-independent framework for identifying geochemical thresholds and separating overlapping populations in complex datasets.

Fuzzy c-means (FCM) clustering

Fuzzy c-means (FCM) is an unsupervised clustering algorithm originally introduced by Dunn⁴⁰ and further generalized by Bezdek⁴¹. It is designed to partition a dataset into a predefined number of clusters, allowing each data point to belong to multiple clusters simultaneously with varying degrees of membership. Unlike crisp clustering techniques, in which samples are assigned exclusively to a single cluster, FCM represents cluster affiliation in a fuzzy manner⁴¹.

Fuzzy c-means (FCM) is an unsupervised clustering algorithm designed to partition a dataset into a predefined number of clusters while allowing each data point to belong to multiple clusters simultaneously with varying degrees of membership. Unlike crisp clustering techniques, in which samples are exclusively assigned to a single cluster, FCM represents cluster affiliation in a probabilistic-fuzzy manner, thereby explicitly accounting for uncertainty and gradual transitions among clusters^{22,42}.

Prior to FCM analysis, all variables were standardized using Z-score normalization to remove scale effects and ensure comparable contribution of variables to the clustering algorithm. This transformation was used only as a preprocessing step for multivariate clustering and not for direct geochemical interpretation. Each sample value was standardized using the Z-score transformation, in which the mean of the variable was subtracted from the observed value and the result was divided by the standard deviation of that variable.

The algorithm seeks to identify optimal cluster centers by minimizing a weighted objective function that measures the total within-cluster variance. This objective function is formally expressed as:

$$J_m = \sum_{i=1}^N \sum_{j=1}^k u_{ij}^m \|x_i - v_j\|^2 \quad (2)$$

where N denotes the number of data points, k is the number of clusters, x_i represents the i -th data vector, and v_j is the centroid of cluster j . The membership coefficient u_{ij} indicates the degree to which data point i belongs to cluster j , subject to the constraint that the sum of memberships for each data point across all clusters equals one. The fuzzification exponent $m > 1$ controls the level of cluster overlap, with higher values producing more diffuse and overlapping cluster boundaries^{43,44}.

The optimization process proceeds iteratively through alternating updates of membership degrees and cluster centers until convergence criteria are satisfied, typically defined by a maximum number of iterations or a minimum change in the objective function or membership matrix. This iterative scheme enables FCM to effectively capture complex data structures and smooth transitions between clusters without imposing strict separation boundaries. One of the principal advantages of FCM lies in its ability to model ambiguity and partial membership, which is particularly valuable in datasets characterized by overlapping distributions or gradual pattern changes. The resulting membership matrix provides richer information than hard assignments, allowing for the assessment

of clustering confidence and internal structure. Furthermore, FCM does not require assumptions regarding data normality and can be applied to high-dimensional datasets after appropriate normalization^{25,29}.

Despite its strengths, FCM is sensitive to the choice of the number of clusters, initial cluster centers, and the fuzzification parameter. Therefore, validation indices and repeated runs with different initializations are commonly employed to enhance clustering robustness and stability³⁷ (Xie & Beni, 2002). Nonetheless, owing to its conceptual simplicity, mathematical transparency, and flexibility, FCM remains one of the most widely adopted fuzzy clustering algorithms in pattern recognition and data analysis.

To determine the optimal number of clusters in the FCM analysis, four complementary cluster validity indices were calculated: the Fuzzy Partition Coefficient (FPC), the Davies–Bouldin Index (DB), the Silhouette Index, and the Xie–Beni (XB) index. FPC measures the overall fuzziness of the partition, with higher values indicating a clearer cluster structure. The DB index evaluates the ratio of within-cluster compactness to between-cluster separation, where lower values indicate better clustering. The Silhouette index assesses the degree of separation between clusters, with higher values corresponding to better-defined clusters. The XB index combines cluster compactness and separation, and lower values indicate a more optimal partition. In this study, these indices were examined across different values of K to identify the most statistically robust and geologically meaningful clustering solution.

Integrated Fractal–FCM Anomaly Index (IFFAI)

To integrate the anomaly intensity of the target element with the multielement geochemical affinity obtained from clustering, an Integrated Fractal–FCM Anomaly Index (IFFAI) was developed. In this framework, the fractal component was derived from the C–N fractal model, in which the fractal threshold was used as the basis for distinguishing anomalous samples from background and for constructing a normalized fractal intensity measure. The fuzzy component was defined from the membership degree of each sample in the anomalous FCM cluster. These two standardized components were then mathematically integrated into a single composite index to identify samples that are both geochemically anomalous in terms of target-element intensity and consistent with the multielement mineralization-related pattern.

In this study, this integrative approach is formalized as the Integrated Fractal–FCM Anomaly Index (IFFAI), in which both the fractal intensity component and the fuzzy membership component are first standardized and subsequently combined using a mathematical composite function. The primary objective of this index is to reduce uncertainty, enhance the signal of genuine anomalies, and improve the accuracy of exploration prioritization, particularly in marginal and transitional zones where individual methods typically show their greatest limitations. In the first step of developing the IFFAI, the geochemical anomaly intensity of the target element (Au) is quantified using thresholds derived from C–N fractal modeling. The purpose of defining the Fractal intensity Index (FII) is to transform raw concentration values into a dimensionless, standardized, and comparable metric within the [0,1] interval that solely reflects anomaly intensity and is independent of data scale. Accordingly, the FII for each sample x is defined in Eq. (3) as follows:

$$FII(x) = \begin{cases} 0 & , \quad C(x) \leq Thr \text{ (Background population)} \\ (C(x) - Min(c)) / (Max(c) - Min(c)) & , \quad C(x) > Thr \text{ (Anomaly population)} \end{cases} \quad (3)$$

where $C(x)$ is the concentration of the target element at location x , and Thr denotes the fractal anomaly threshold.

The FII defined in this study exhibits several key characteristics that make it a suitable component for the development of the integrated IFFAI. First, this index is entirely data-driven, in that the fractal threshold is extracted exclusively from the self-similar behavior of the data and slope breakpoints in the logarithmic C–N plot, without adopting any prior distributional assumptions. Second, by explicitly assigning zero values to FII for samples falling below the fractal threshold, the geochemical background is systematically removed, thereby preventing the artificial amplification of insignificant fluctuations. Third, by normalizing anomaly intensity to the [0, 1] range,

FII is transformed into a standardized, dimensionless metric that enables direct comparison among samples and straightforward combination with other indices. Finally, the comparable scale of this index with the fuzzy membership degrees derived from FCM clustering ensures a transparent and mathematically consistent integration, providing the necessary framework for uncertainty reduction and anomaly signal enhancement within the IFFAI. To integrate the fractal and FCM results, an additional index based on the FCM method is defined. The values of the FCM Membership Degrees Index (MDI) for each sample are taken as the membership degree of that sample in the anomalous cluster. This value ranges between 0 and 1, thereby allowing direct integration with the fractal-based FII. Subsequently, the integrated index is calculated based on the following relationship:

$$\text{IFFAI} = (\text{FII}(x) + \text{MDI}(x))/2 \quad (4)$$

Results

Fractal modeling of Au

In this study, the C–N fractal method was employed to discriminate geochemical populations of gold and to determine the thresholds associated with background and anomalous zones. This method, which is based on the scale-invariant behavior of the spatial distribution of geochemical elements, enables the identification of independent geochemical regimes through changes in the slope of the logarithmic relationship between concentration and cumulative area.

However, one of the fundamental challenges in applying the C–N fractal approach is the determination of the true and optimal number of fractal classes, as well as avoiding over-segmentation of the data. In many previous studies, this issue has commonly been addressed through visual inspection of log–log plots, leading to a degree of subjectivity in class selection. To reduce this subjectivity and enhance the statistical robustness of the results, the Bayesian Information Criterion (BIC) was adopted in this study to determine the optimal number of geochemical populations.

The BIC, originally introduced by Schwarz⁴⁵, is a Bayesian model selection criterion that establishes a balance between model goodness-of-fit and model complexity. By explicitly penalizing increases in the number of model parameters, BIC effectively mitigates the risk of overfitting, a property that makes it particularly suitable for fractal analyses with a high potential for excessive class partitioning⁴⁶.

Within the framework of the C–N fractal method, the relationship between gold concentration and the cumulative area characterized by concentrations exceeding a given threshold is modeled as a piecewise linear relationship in logarithmic space. Each linear segment represents an independent fractal regime or geochemical population, which is associated with changes in the geological processes controlling gold concentration. In this study, for each assumed number of fractal classes, a piecewise linear regression was fitted to the log–log C–N plot, and the residual sum of squares (RSS) was calculated. Subsequently, the BIC value was computed according to the following equation:

$$\text{BIC} = n \ln \left(\frac{\text{RSS}}{n} \right) + k \ln(n) \quad (5)$$

where n denotes the number of data points and k represents the number of model parameters, which is proportional to the number of fractal classes^{45,46}.

The results indicate that, although the residual sum of squares (RSS) decreases with an increasing number of fractal classes, the BIC value decreases only up to a distinct minimum and then increases thereafter. This behavior suggests that adding more fractal classes does not necessarily reflect the presence of real geochemical populations, but may instead result from fitting statistical noise in the data. In this study, the minimum BIC value identifies a specific number of fractal classes for gold, which is interpreted as the optimal number of geochemical

Fig. 4 BIC-based selection of the optimal number of piecewise linear segments representing fractal geochemical populations in the C–N model.

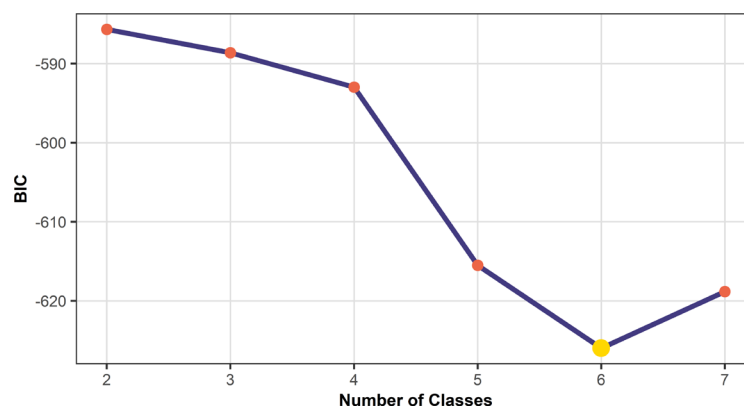
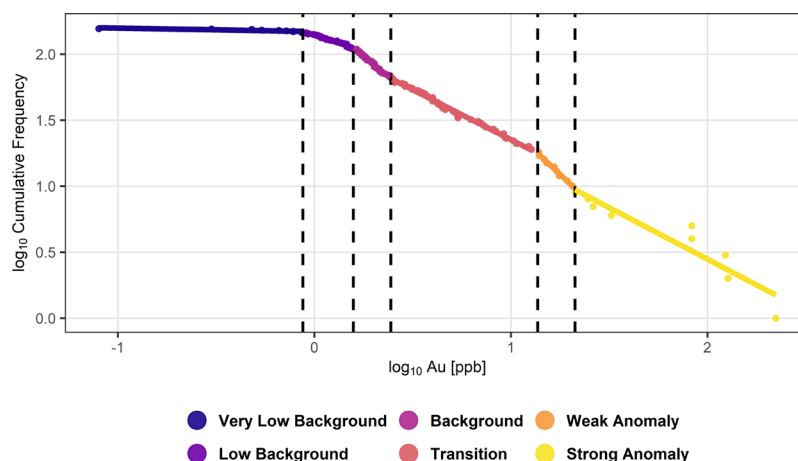


Fig. 5 Log–log C–N fractal plot of Au showing scaling regimes and breakpoint-derived geochemical classes in the Saqqez area.



populations (Fig. 4). This optimal class number shows good consistency with the regional geological framework and established metallogenic models, enabling a meaningful discrimination between background zones and anomalies of varying intensities.

From an analytical perspective, integrating the BIC with the C–N fractal method represents an important step toward standardizing the process of geochemical population separation. This approach not only preserves the inherent multiscale and fractal characteristics of geochemical data, as emphasized by Cheng⁷, but also incorporates principles of statistical model selection to prevent exaggerated and unrealistic interpretations. Moreover, as noted by Konishi and Kitagawa⁴⁷, information-theoretic criteria such as BIC provide powerful tools for identifying the optimal level of model complexity, particularly in problems where the true underlying structure of the data is unknown.

Overall, the results of this study demonstrate that applying the Bayesian Information Criterion within C–N fractal analysis of gold provides an objective, reproducible, and statistically robust framework for determining the optimal number of geochemical populations, thereby significantly enhancing the reliability of geological interpretations derived from fractal analyses. In this study, the BIC curve indicates that the optimal number of fractal classes is six, based on which the most appropriate breakpoints in the logarithmic fractal plot were determined.

Figure 5 presents the log–log plot of C–N fractal model. According to multifractal theory, linear segments in such plots indicate scale-invariant geochemical populations, whereas changes in slope reflect transitions between geochemically distinct regimes governed by different controlling processes^{7,12}.

The C–N plot of Au in the Saqqez area exhibits a clear piecewise linear structure, characterized by multiple well-defined slope breakpoints. These breakpoints objectively indicate the presence of several independent geochemical populations rather than a single homogeneous background. Importantly, the identification of these

breakpoints is entirely data-driven and does not rely on predefined thresholds, distributional assumptions, or external statistical criteria.

Based on the position and continuity of slope changes, the gold dataset was subdivided into six distinct fractal classes, representing progressively increasing concentration levels. These classes include three background populations (very low background, low background, and background), one transition population, and two anomalous populations classified as weak anomaly and strong anomaly.

The three background classes display relatively gentle slopes and high cumulative frequencies, reflecting widespread, low-grade Au concentrations that are primarily controlled by lithological variability and regional geochemical heterogeneity. The transition class marks a critical geochemical threshold at which the influence of mineralization-related processes begins to exceed that of background controls, as evidenced by a noticeable change in the slope of the C–N curve.

The weak and strong anomaly classes are distinguished by markedly steeper slopes and rapidly decreasing cumulative frequencies, representing increasingly rare but significantly elevated Au concentrations. Such behavior is characteristic of geochemical populations directly associated with ore-forming processes and is typically linked to structurally controlled mineralization zones, fault systems, vein networks, and permeable lithological domains.

It is noteworthy that the number of fractal classes and the corresponding threshold values were derived solely from the intrinsic scaling behavior of the data, as revealed by slope breakpoints in the C–N plot. No arbitrary cut-off values or prior geological constraints were imposed during this process. Consequently, the resulting classification provides an objective and reproducible framework for distinguishing background, transition, and anomalous gold populations.

Table 2 summarizes the results of the fractal C–N modeling applied to Au concentrations in the study area and presents the quantitative characteristics of the six identified fractal populations. For each class, the corresponding lower threshold value, fractal dimension, number of samples, and concentration range are reported, providing a comprehensive description of the internal scaling behavior and population structure of gold.

The six identified fractal classes form the quantitative basis for normalizing gold concentrations and for constructing the intensity component of the integrated Fractal–FCM anomaly index (IFFAI). By capturing the inherent multi-scale nature of gold distribution, this fractal classification plays a pivotal role in enhancing anomaly discrimination and in ensuring a consistent linkage between geochemical intensity and subsequent multivariate fuzzy-clustering results.

Figure 6 illustrates the statistical behavior of gold concentrations across the fractal classes derived from the C–N fractal analysis. Panel A presents the kernel density distributions of log-transformed Au concentrations for the six identified fractal classes, while Panel B summarizes the same classes using boxplots plotted on a logarithmic scale to highlight contrasts in central tendency and dispersion. As shown in Fig. 6A, the very low background, low background, and background classes exhibit relatively narrow and partially overlapping density distributions concentrated at low log (Au) values, reflecting a stable geochemical background with limited variability. In contrast, the transition class displays a broader and more asymmetrical distribution, indicating increased variability and marking a geochemical boundary zone where background conditions progressively evolve into anomalous behavior. The weak anomaly and strong anomaly classes demonstrate a pronounced rightward shift in their

Table 2 Summary of fractal classes, threshold concentrations, fractal dimensions, and sample distribution for gold.

Class	Threshold from(ppb)	Fractal dimension	Samples	Range (ppb)
Very low background	0	0.026	11	0.08–0.86
Low background	0.874	0.482	36	0.91–1.5
Background	1.579	1.232	43	1.64–2.4
Transition	2.449	0.757	48	2.49–12.7
Weak anomaly	13.685	1.42	9	13.9–20.5
Strong anomaly	21.19	0.78	9	21.19–194

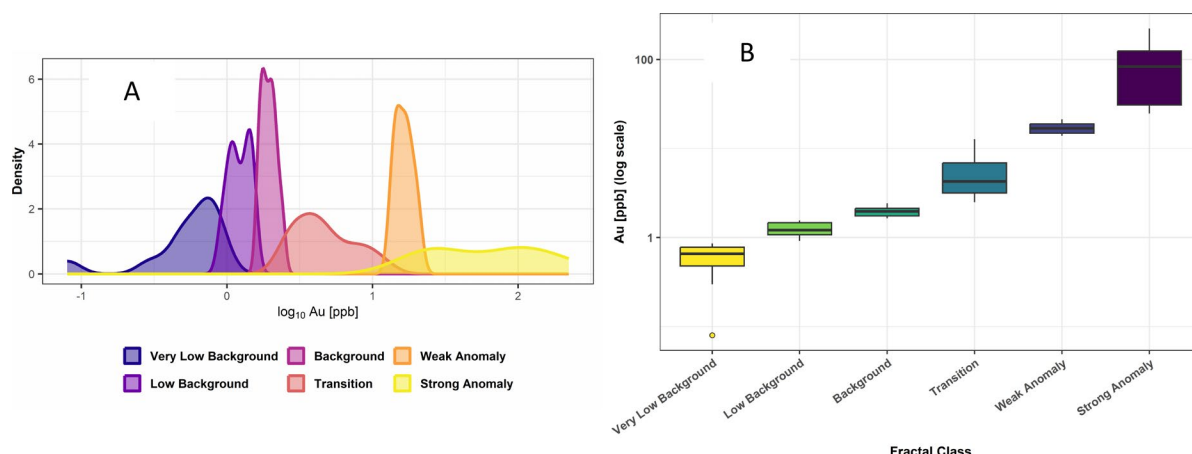


Fig. 6 (A) Kernel density distributions and (B) boxplots of log-transformed gold concentrations across fractal classes derived from the C–N model.

density curves, accompanied by a clear reduction in overlap with background populations, which signifies the emergence of geochemically distinct gold-enriched regimes controlled by mineralization processes.

The boxplots shown in Fig. 6B further confirm these observations by providing a robust statistical comparison of gold concentrations among the fractal classes. A systematic increase in median values and interquartile ranges is evident from background to anomalous classes, indicating a consistent intensification of gold enrichment. Background-related classes are characterized by low medians and limited dispersion, whereas anomalous classes exhibit higher medians and wider ranges, reflecting both elevated concentrations and greater heterogeneity. The Transition class presents intermediate statistical properties, supporting its interpretation as a mixing zone influenced by both background dispersion and localized mineralizing inputs.

Together, the density and boxplot analyses demonstrate that the fractal thresholds derived from the C–N model effectively separate gold populations with distinct statistical characteristics. This consistency between fractal segmentation and empirical data distributions confirms that the identified classes are not arbitrary numerical divisions but represent meaningful geochemical populations. Consequently, the results shown in Fig. 6 provide a robust statistical foundation for the normalization of gold concentrations and justify their subsequent integration with fuzzy membership degrees within the proposed IFFAI.

Identification of mineralized geochemical signatures using FCM

The FCM clustering algorithm was applied to the entire geochemical dataset consisting of 21 variables, normalized using the Z-score transformation. To discriminate geochemical populations associated with mineralization from the regional background in the Saqqez area, the FCM clustering algorithm was applied to geochemical datasets normalized using the Z-score transformation. This normalization effectively removes scale dependency among elements and ensures their equal contribution to the clustering process. The use of FCM is particularly advantageous in complex exploration environments because it assigns continuous membership values to each sample, thereby explicitly accounting for uncertainty and overlapping geochemical populations. To determine the optimal number of clusters, several cluster-validity indices, including the Fuzzy Partition Coefficient (FPC), Davies–Bouldin Index (DB), Silhouette Index, and Xie–Beni (XB) Index, were calculated for varying values of K. The behavior of these indices is illustrated in Fig. 7. At K=2, the FPC reaches its maximum value, indicating the highest degree of fuzzy separation between clusters. Simultaneously, the XB index attains its minimum value at this cluster number, reflecting the most favorable balance between intra-cluster compactness and inter-cluster separation.

Fig. 7 Cluster validity indices (FPC, Davies–Bouldin, Silhouette, and Xie–Beni) used to determine the optimal number of clusters in the fuzzy c-means (FCM) analysis.

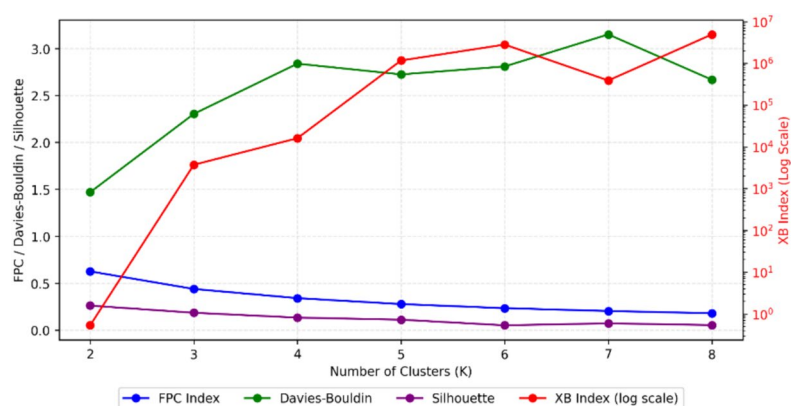


Fig. 8 Mean Z-score profiles of the two FCM clusters showing relative positive and negative elemental contributions within each cluster.

Although increasing the number of clusters yields minor local improvements in the DB and Silhouette indices, these gains are accompanied by increased model complexity and reduced geological interpretability. Therefore, $K=2$ was selected as a parsimonious and geology-oriented solution that robustly distinguishes the primary geochemical populations corresponding to mineralized and background environments.

The geochemical characteristics of the two clusters were further examined by analyzing the mean Z-scores of individual elements (Fig. 8). This figure illustrates the top positive and negative contributors within each cluster, highlighting their relative enrichment or depletion with respect to the overall dataset. Cluster 2 is characterized by comparatively higher relative contributions from Au, Bi, Sb, Hg, and Sn, which collectively represent a

diagnostic elemental association for gold-related mineralization in the study area. Although the absolute mean Z-scores of some of these elements remain slightly negative, they are internally enriched relative to other elements within the same cluster and therefore play a key discriminative role. This geochemical signature is consistent with shear-zone-hosted gold mineralization and polymetallic vein systems spatially associated with major fault zones and metamorphic host rocks. In contrast, Cluster 1 exhibits relatively higher mean Z-scores for Zn, V, Mn, Ni, and Pb, accompanied by a depletion in gold-pathfinder elements. This elemental association reflects the regional geochemical background and is predominantly linked to mafic to ultramafic lithological units.

From a geological and metallogenic perspective, the two clusters identified by the FCM algorithm reflect distinct geochemical populations that are consistent with the lithological heterogeneity and mineralization style of the Saqqez area. Cluster 2 is clearly associated with gold mineralization and exhibits a diagnostic elemental assemblage dominated by Au, Bi, Sb, Hg, and Sn. This association is characteristic of orogenic, shear-zone-hosted gold systems formed under compressional tectonic regimes within metamorphic belts. The enrichment of Sb, Bi, and Hg alongside gold indicates the involvement of deeply sourced hydrothermal fluids migrating along major fault zones and ductile–brittle shear structures. Spatially, this cluster corresponds to highly deformed metavolcanic and metasedimentary rocks, including schist and phyllite, where intense mylonitization and pervasive alteration processes such as silicification, sericitization, sulfidization, and carbonatization are well developed. These features are fully consistent with the metallogenic framework of gold mineralization reported in the Sanandaj–Sirjan metamorphic belt and its tectonic junctions.

In contrast, Cluster 1 represents the regional geochemical background and is primarily controlled by lithology rather than hydrothermal mineralization. This cluster is characterized by relatively higher contributions from Ni, V, Mn, and Zn, elements that are typically enriched in mafic to ultramafic rocks and metavolcanic protoliths. The dominance of these elements indicates a strong lithogenic control linked to basaltic–andesitic units and metamorphic rocks derived from mafic compositions. The systematic depletion of gold and its associated pathfinder elements within this cluster suggests limited interaction with mineralizing fluids and a low metallogenic potential. Consequently, Cluster 1 can be interpreted as a background geochemical population reflecting regional metamorphism and primary rock composition, whereas Cluster 2 delineates structurally controlled hydrothermal systems favorable for gold mineralization. The sharp geochemical contrast between these two clusters provides robust geological support for the selection of $K=2$ and confirms the effectiveness of FCM in separating mineralized zones from the regional background in the Saqqez area. The clear contrast in elemental behavior between the two clusters shown in Fig. 8 provides strong geochemical justification for the selection of $K=2$ and confirms the capability of FCM to effectively separate anomalous and background populations.

Beyond its statistical robustness, the selection of $K=2$ is fundamental to the construction of the IFFAI. Specifically, the FCM membership degree of each sample to the mineralization-related cluster was used as a quantitative measure of anomaly confidence and integrated with the fractal-standardized gold intensity. This hybrid strategy enables the simultaneous consideration of anomaly magnitude and attribution uncertainty, leading to anomaly maps with improved clarity, reduced noise, and enhanced exploration relevance compared to single-method approaches.

Integrated fractal–FCM anomaly mapping results

It should be emphasized that the C–N fractal analysis and FCM clustering were not intended to determine the same type of geochemical classes, and thus the identification of six fractal classes versus two fuzzy clusters ($K=2$) does not represent a methodological contradiction. In our framework, the C–N fractal analysis was applied strictly to the univariate Au distribution to model concentration scaling and establish data-driven geochemical populations and threshold values. The resulting six fractal classes represent successive concentration regimes within the gold dataset, which are used to quantify the spatial intensity of the Au anomalies. In contrast, FCM clustering was employed as a multivariate fuzzy partitioning step to distinguish samples associated with the broader anomalous geochemical pattern from the barren background based on combined elemental associations. For this multivariate

purpose, cluster validity indices identified the two-cluster solution as optimal: one background cluster and one anomalous cluster. Therefore, FCM was not designed to reproduce or inherit the six fractal classes of gold, but rather to quantify each sample's degree of membership in the anomalous multi-element cluster. The role of C–N fractal analysis is thus to provide objective anomaly thresholds and intensity classes, whereas the role of FCM is to provide soft membership degrees for the multi-element signature, with both complementary outputs ultimately integrated into the IFFAI.

Figure 9 illustrates the spatial distribution of sample membership degrees within the anomalous cluster extracted using FCM clustering (Cluster 2). In this map, higher membership values indicating a stronger geochemical similarity of samples to the multielement anomalous pattern associated with gold mineralization are clearly highlighted. Conversely, lower membership values, which are indicative of geochemical background zones or areas lacking a multielement mineralization signature, are also delineated.

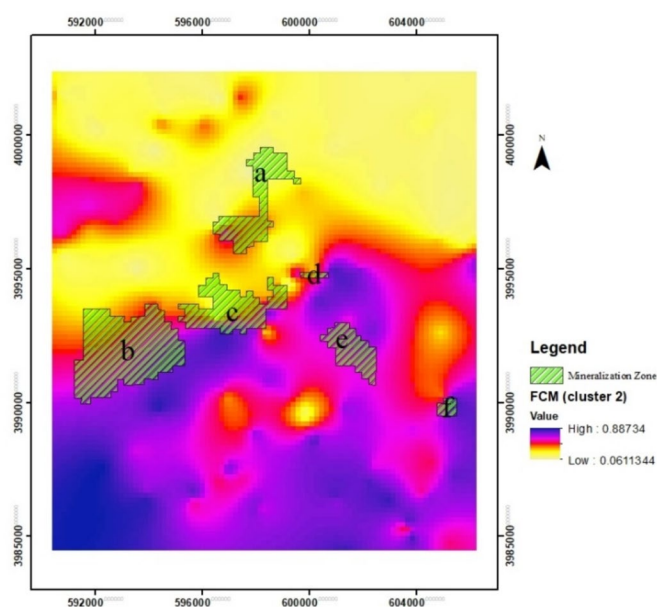
The C–N fractal analysis of gold concentrations yielded an anomaly threshold of 13.68 ppb. This threshold was used together with the multielement degree of inclusion (MDI) to calculate the final integrated fractal–FCM anomaly index (IFFAI).

Figure 10 presents the final anomaly maps derived from the fractal classes, binary fractal thresholding, continuous IFFAI, and classified IFFAI. The integrated maps delineate more compact anomaly zones and provide a clearer representation of the target areas for follow-up exploration.

Discussion

The results obtained in this study indicate that the identification of geochemical anomalies in structurally complex tectonic environments particularly in shear zone hosted gold systems is inherently associated with uncertainty and overlap between background and mineralized populations. Non-normal data distributions, the presence of multiple geochemical populations, the influence of lithology and alteration processes on element dispersion, and the role of pathfinder elements with contrasting geochemical behaviors collectively imply that no single method whether based on univariate thresholding or multivariate technique can independently achieve a complete and reliable delineation of all mineralized zones. Consequently, the analysis demonstrates that an exclusive focus on anomaly intensity or, alternatively, a sole emphasis on multivariate geochemical patterns may both lead to

Fig. 9 Spatial distribution of fuzzy c-means (FCM) membership degrees for the anomalous geochemical cluster (Cluster 2).



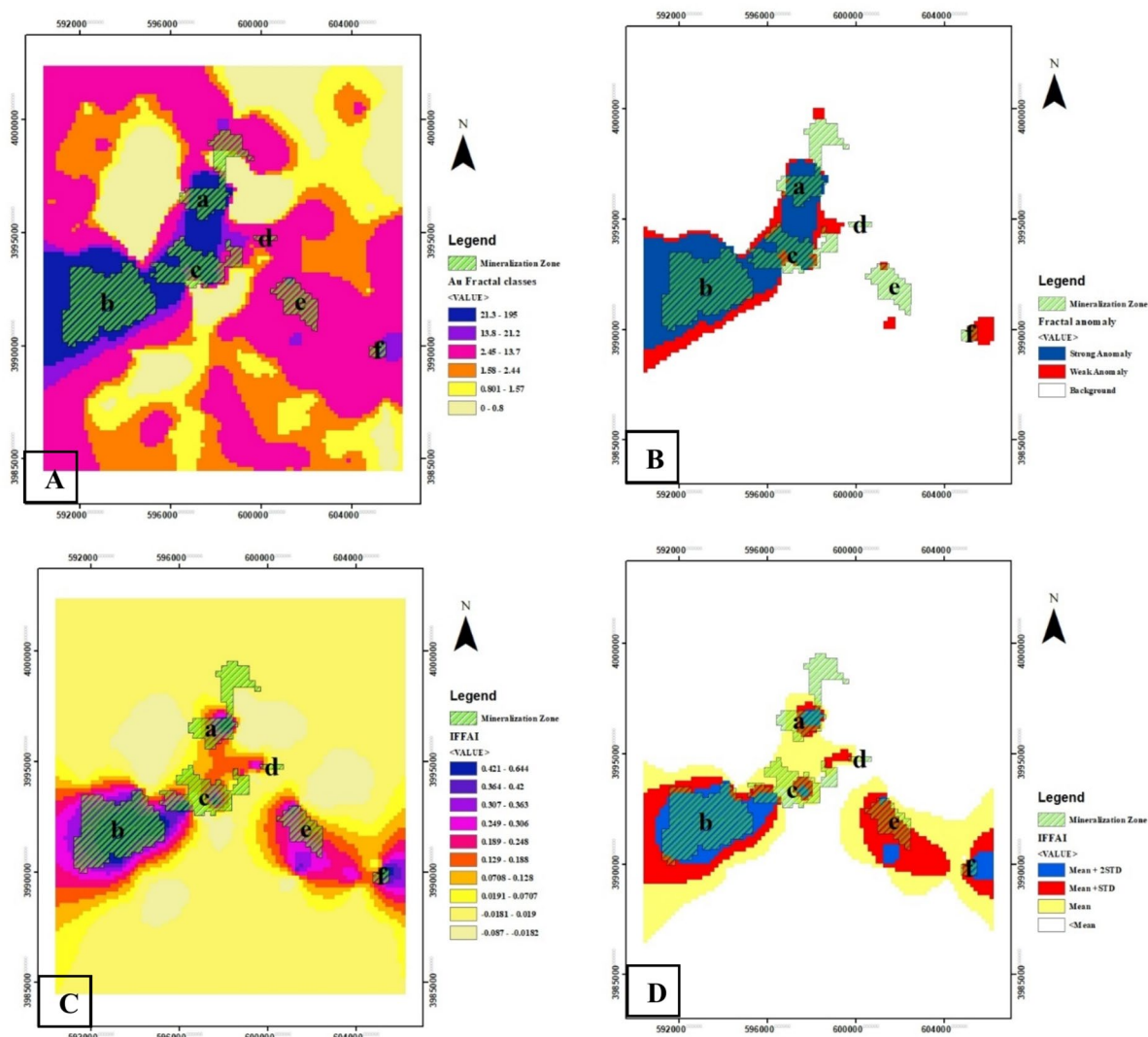


Fig. 10 Comparison of Au geochemical anomaly maps derived from fractal modeling and the integrated fuzzy–fractal index (IFFAI), (A) Fractal class map of Au concentrations, (B) Classified fractal anomaly map, (C) Continuous IFFAI map, (D) Classified IFFAI map.

incomplete or unstable interpretations. This conceptual limitation underscores the necessity of integrated frameworks capable of simultaneously accounting for anomaly intensity and the degree of confidence associated with sample attribution to mineralized environments. Accordingly, the following section introduces the Integrated Fractal–FCM Anomaly Index (IFFAI), which has been developed to reduce uncertainty, enhance anomaly signals, and improve exploration prioritization.

Integrated Fractal-FCM Anomaly Index (IFFAI)

The spatial overlap between portions of the high-membership zones and several of the six known mineralized zones in the study area demonstrates the effectiveness of the FCM method in identifying geochemical patterns related to mineralization and confirms the general consistency of its results with independent geological evidence (Fig. 9). These known mineralized zones are therefore used as validation criteria for assessing the reliability and certainty of the obtained results. Nevertheless, a detailed examination of the map reveals that the FCM method

alone is not capable of fully identifying all known mineralized zones in the region. Some recognized mineral zones exhibit only moderate membership values, while others are entirely located within areas characterized by low membership degrees. This behavior primarily arises from the multivariate and relative nature of the FCM approach, in which clustering is more sensitive to the relative interrelationships among geochemical elements than to the absolute concentration of the target element (Au).

In complex geological environments, such as shear-zone hosted gold systems, lithological variability, localized hydrothermal alteration, geochemical dilution effects, and the influence of pathfinder elements may cause certain genuinely mineralized zones to deviate from the dominant multielement anomalous pattern. Consequently, these zones may not exhibit a complete overlap with the anomalous FCM cluster. As a result, exclusive reliance on FCM membership degrees may lead to the omission or attenuation of real geochemical anomalies.

This limitation indicates that, although FCM is a powerful tool for identifying geochemical patterns and modeling uncertainty, it is insufficient on its own to represent the true intensity of anomalies and to fully discriminate mineralized zones. Therefore, to overcome this shortcoming, it is necessary to explicitly incorporate anomaly intensity information directly derived from the concentration of the target element into the decision-making process in a data-driven manner.

From this perspective, integrating the FCM clustering with fractal modeling of gold concentrations allows for the simultaneous consideration of anomaly intensity (based on fractal thresholds) and the confidence of sample assignment to mineralized environments (based on FCM membership degrees). Accordingly, locations that satisfy both criteria concurrently can be identified as high-priority exploration anomalies with greater confidence and reliability.

Based on the results of the C–N fractal modeling, the gold geochemical data in the study area comprise several independent populations, each of which is characterized by a distinct linear segment in log–log space. The threshold (Thr) represents the boundary marking the onset of anomalous populations. According to the fractal results, the threshold value separating anomalous gold populations was determined to be 13.68 ppb.

To integrate the fractal and FCM results, the values of the MDI for each sample are taken as the membership degree of that sample in the anomalous cluster (Cluster 2). The MDI values are directly integrated with the fractal-based FII. To prevent the enhancement of spurious anomalies caused by local fluctuations or data noise, an auxiliary constraint derived from the FCM clustering results is applied. Accordingly, samples with membership degrees lower than 0.2 in the anomalous cluster are excluded from the index computation, and their MDI values are set to zero.

This filtering step ensures that only samples that are not only significant in terms of fractal intensity, but also consistent with mineralization-related multielement geochemical patterns, are retained for subsequent analysis. Thus, the role of FCM at this stage is to increase exploration confidence and to suppress the influence of geochemical responses unrelated to mineralization. Subsequently, the integrated index of IFFAI was calculated based on the proposed equations.

Figure 10 presents an integrated comparison of four different approaches for gold geochemical anomaly detection, displayed as maps a to d. This comparison aims to evaluate the capability of each method to delineate anomalies, enhance spatial concentration, and improve exploration prioritization, as well as to assess their spatial agreement with six known mineralization zones (a, b, c, d, e, and f). These zones, defined based on field evidence such as gold-bearing veins and mineralized structures, serve as independent geological references for validating the robustness and reliability of the derived anomalies.

Map A illustrates the spatial distribution of fractal classes of gold obtained from the C–N fractal model. In this map, the dataset is subdivided into background, transition, weak anomaly, and strong anomaly classes, reflecting relative variations in gold concentration. Although this map effectively captures regional-scale variations in gold abundance, it exhibits a widespread and diffuse pattern of anomalies with limited spatial coherence. Portions of zones b and c are highlighted within anomalous classes; however, the delineation of zone a remains incomplete, while zones d and e are almost entirely undetected, and zone f is only partially represented. This pattern indicates that univariate fractal modeling, despite its strength in anomaly intensity extraction, is insufficient for reliably

distinguishing mineralization-related anomalies from non-genetic geochemical enrichments, resulting in limited geological interpretability and variable anomaly quality.

Map B represents the classified fractal anomaly map, derived by applying the fractal threshold to distinguish background, weak, and strong anomalies. Compared to Map A, this representation exhibits improved focusing on higher-concentration gold anomalies and reduces the extent of background regions. Nevertheless, the anomaly pattern remains heterogeneous, with several strong anomalies appearing in areas lacking known mineralization, while significant portions of true mineralized zones particularly zones d, e, and parts of f are either weakly expressed or completely omitted. This observation demonstrates that exceeding a fractal threshold alone does not guarantee geological significance or anomaly reliability.

In contrast, Map C depicts the continuous IFFAI, generated by combining fractal-based FI with fuzzy membership degrees to the anomalous cluster derived from FCM (MDI). In this map, the spatial configuration of anomalies changes fundamentally: high-value anomaly cores exhibit strong spatial continuity, improved focus, and a clear correspondence with known mineralization zones. Nearly all zones a through f are coherently delineated, and anomalous areas are largely confined to regions supported by geological evidence. The overall quality of anomalies is substantially enhanced, with both intensity and geological credibility reinforced, while spurious anomalies are markedly suppressed. The results indicate that IFFAI successfully identifies all known mineralized areas. Finally, Map D displays the classified IFFAI map, segmented using statistical criteria (mean and standard deviation (STD)), which represents the most practical product for exploration decision-making. Zones with IFFAI values greater than $\text{Mean} + \text{STD}$ are identified as high-confidence and high-priority targets. These areas show the strongest spatial overlap with mineralization zones a–f and are characterized by compact, coherent, and clearly defined anomaly patterns. Background regions are effectively eliminated, reflecting a significant improvement in the signal-to-noise ratio relative to individual methods.

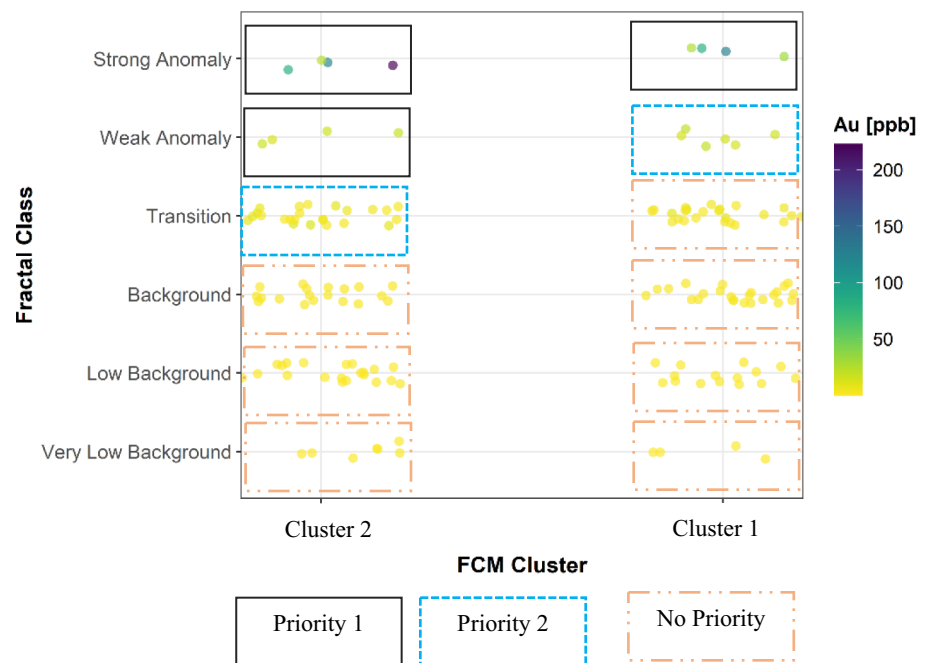
The comparison between Maps A and B versus Maps C and D demonstrates that fractal-based methods alone, despite their data-driven nature and effectiveness in detecting concentration intensity, are unable to fully and accurately delineate mineralized zones. Similarly, as shown previously, the standalone FCM approach fails to capture several known mineralization zones. The integration of fractal modeling and fuzzy clustering within the IFFAI framework effectively resolves these limitations by simultaneously enhancing anomaly intensity and geological confidence. Consequently, all known mineralization zones are detected with high quality, and the process of anomaly delineation and exploration targeting is significantly improved.

Prioritizing geochemical samples using hybrid FCM-fractal approach

Following the delineation of geochemical anomalies by means of fractal modeling and the identification of multivariate geochemical patterns through fuzzy clustering, the subsequent critical step in the exploration workflow is the prioritization of geochemical samples based on their exploration significance. At this stage, anomaly intensity derived from fractal classes is jointly evaluated with the geological reliability of sample attribution to the mineralization-related cluster inferred from the FCM algorithm, aiming to distinguish high-potential targets from transitional and background samples.

Figure 11 illustrates the distribution of gold geochemical samples according to fractal classes versus FCM clusters. The color scale represents gold concentrations (ppb), enabling a simultaneous assessment of anomaly intensity and cluster affiliation for each sample. Based on the classification scheme shown in Fig. 11, all samples assigned to the strong anomaly class by fractal modeling are classified as Priority 1, regardless of whether they belong to FCM Cluster 1 or Cluster 2. The rationale behind this classification is that strong fractal anomalies correspond to significantly elevated gold concentrations, which alone indicate high exploration potential and therefore warrant immediate follow-up. In addition, samples classified as weak anomalies by the fractal method and simultaneously assigned to FCM Cluster 2 are also categorized as Priority 1. As Cluster 2 has been previously interpreted as being genetically associated with gold mineralization characterized by the enrichment of Au

Fig. 11 Exploration-based prioritization of geochemical samples derived from fractal anomaly intensity and FCM clustering results



together with pathfinder elements such as Bi, Sb, Hg, and Sn the coincidence of even moderate fractal anomalies with this mineralization-related cluster substantially increases exploration confidence.

Conversely, samples that fall within the weak anomaly fractal class but are assigned to FCM Cluster 1 are considered Priority 2 targets. Although these samples exhibit gold concentrations above background levels, their lack of association with the mineralization-related cluster reduces geological confidence, placing them in a secondary exploration category that requires cautious follow-up. Furthermore, samples belonging to the transitional fractal class and assigned to FCM Cluster 2 are also classified as Priority 2. These samples reflect areas where multivariate geochemical signatures and structural geological conditions are favorable for mineralization, yet the intensity of gold enrichment has not reached the threshold of weak or strong anomalies. Finally, all remaining samples namely those classified within the background, low background, and very low background fractal classes, as well as samples lacking both significant anomaly intensity and association with the mineralization-related cluster are categorized as No Priority samples, as they are unlikely to represent economically meaningful mineralization targets.

Figure 11 clearly demonstrates that integrating fractal anomaly intensity with FCM-based cluster information provides a robust and systematic framework for geochemical sample prioritization. This hybrid FCM–Fractal approach effectively reduces interpretational uncertainty, prevents the premature exclusion of potentially significant samples, and enhances the efficiency of exploration decision-making by guiding follow-up studies toward the most promising targets.

Figure 12 presents the final exploration prioritization map of gold geochemical samples in the study area, based on the results obtained from the integrated fractal–fuzzy clustering framework. This map represents the spatial implementation of the prioritization logic derived from Fig. 11 and illustrates how the combination of gold anomaly intensity, as determined by fractal modeling, with the confidence level of sample assignment to the FCM mineralization cluster can lead to the generation of a decision-support spatial product at the exploration scale. In this map, the samples are classified into three main categories Priority 1, Priority 2, and No Priority each reflecting a different level of exploration significance and certainty. The spatial distribution pattern of Priority 1 samples indicates that they are predominantly concentrated within or in close proximity to the identified mineralization zones, showing a strong spatial correspondence with the known mineralized areas of the region. This spatial

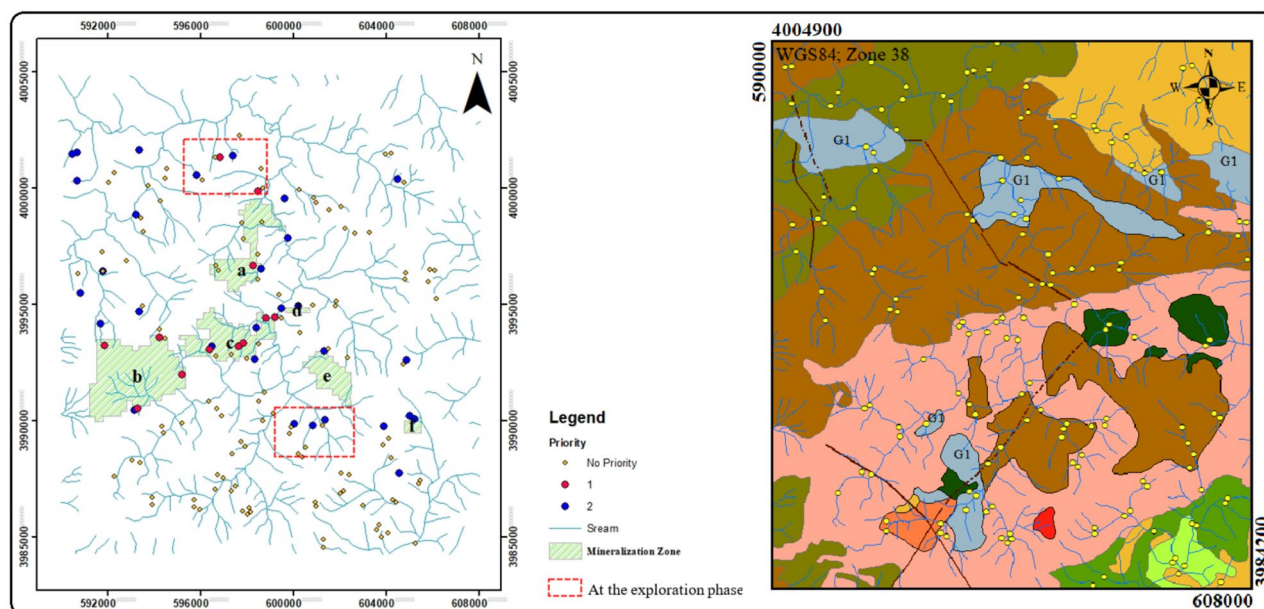


Fig. 12 Spatial distribution of exploration priorities for gold geochemical samples generated using the integrated fractal–FCM approach, showing the relationship between priority classes, known mineralized zones, and areas currently under active exploration.

overlap demonstrates that the proposed integrated framework has been highly effective in accurately identifying samples that simultaneously exhibit high fractal anomaly intensity and multielement geochemical signatures consistent with mineralization. Such a focused concentration of high-priority samples not only confirms the spatial validity of the method but also indicates that the proposed approach successfully reduces the inherent uncertainties of single-method techniques and prevents the random dispersion of geochemical anomalies.

In contrast, Priority 2 samples display a more dispersed spatial pattern across the study area and are commonly located along the margins of mineralized zones or aligned with drainage networks and geological structures. These samples generally exhibit either weaker fractal anomaly intensities or, although they belong to the FCM mineralization cluster in terms of multielement geochemical patterns, their gold concentrations have not yet reached the threshold of strong anomalies. The occurrence of this group around known mineralized areas may reflect lateral mineralization dispersion, erosion and sediment transport effects, or the presence of more subtle or concealed mineralization zones that require targeted follow-up investigations.

No-Priority samples, which constitute a substantial portion of the dataset, are mainly distributed in areas distant from mineralization zones, and their spatial pattern represents the regional geochemical background. These samples do not provide compelling evidence for further exploration, either in terms of fractal anomaly intensity or membership in the FCM mineralization cluster. Excluding them from subsequent exploration stages allows for a more efficient allocation of resources toward targets with higher exploration certainty. The areas delineated by dashed outlines in Fig. 12 correspond to zones that are currently under active exploration and are regarded, from an operational perspective, as principal exploration priorities within the study area. These zones represent key targets and, based on prior geological and geochemical evidence, are likely locations where exploration activities such as channel sampling and detailed field investigations have been conducted or are presently ongoing. The concentration of high-priority samples particularly Priority 1 within these zones indicates that the outcomes of the integrated fractal–FCM framework are not only consistent with the current exploration strategy in the area but also provide a robust quantitative support for validating and reinforcing existing field-based decisions. In this context, these zones play a critical role in the traditional, ground-truth validation of the model results, and their strong spatial coincidence with the outputs of the proposed integrated approach highlights the high capability of

the fractal–FCM integration scenario in reproducing practical exploration logic and ensuring consistency with field observations and previous exploration experiences.

The prioritized geochemical sample map demonstrates that transferring the results of combined fractal modeling and fuzzy clustering into the spatial domain leads to the production of a practical, interpretable, and decision-oriented prioritization map. By reducing noise, enhancing spatial concordance with geological realities, and enabling a staged discrimination of exploration targets, this map can serve as an effective tool for designing follow-up sampling programs, selecting trench locations, and prioritizing exploratory drilling, ultimately contributing to improved efficiency and reduced costs in mineral exploration operations.

The combined fuzzy–fractal approach integrates two complementary sources of information. The C–N fractal model identifies Au anomaly intensity using data-driven thresholds but may highlight enrichments unrelated to mineralization because it is essentially univariate. Conversely, FCM captures multielement geochemical associations and sample-membership uncertainty, although it may not fully delineate all known mineralized zones when used alone. By integrating fractal intensity with FCM membership in the mineralization-related cluster, IFFAI emphasizes samples that are both Au-enriched and geochemically consistent with mineralization. Consequently, the IFFAI map produces more coherent and focused anomalies, suppresses geologically unsupported responses, and improves correspondence with the field-defined mineralization zones. Thus, its advantage lies in combining anomaly intensity and multielement geological affinity within a single targeting index.

Conclusion

In this study, an innovative integrative framework was proposed for the identification, validation, and prioritization of gold geochemical anomalies based on the synergistic integration of fractal modeling and FCM clustering. The results obtained from the Saqqez area demonstrate that standalone approaches including univariate fractal modeling and multivariate clustering applied independently each suffer from inherent limitations in fully representing the geochemical complexity of the study area. Fractal methods are primarily effective in discriminating anomaly intensity but exhibit limited sensitivity to multielement geochemical patterns and uncertainty in sample attribution. Conversely, although FCM clustering shows a strong capability for identifying genetically meaningful mineralization patterns, it is not, by itself, sufficient for adequately differentiating anomaly intensity or completely suppressing background geochemical noise.

To address these limitations, the IFFAI was developed as a continuous metric for the simultaneous evaluation of anomaly intensity and geochemical attribution confidence. By normalizing the fractal-based gold anomaly intensity and integrating it with the FCM membership degree of samples within the mineralization-related cluster, IFFAI provides a scalable and interpretable index ranging from 0 to 1. Within the context of this study, IFFAI primarily served as a supporting analytical tool to enhance spatial analysis, reduce noise effects, and assess the continuity and coherence of anomalous patterns. Comparative analysis indicates that the spatial distribution of high IFFAI values exhibits a strong overlap with known mineralized zones and delivers a more coherent representation of mineralized areas than individual methods.

Nevertheless, the final exploration priority map was derived directly from the integrated interpretation of fractal modeling results (anomaly intensity classes) and the FCM-derived cluster assignments, rather than from the direct thresholding of the IFFAI values. Within this framework, the prioritization logic was defined such that samples characterized by high fractal anomaly intensity and strong geochemical affinity with the mineralization-related FCM cluster were classified as Priority 1, whereas weaker or transitional combinations of these two criteria were assigned to Priority 2. Samples lacking concurrent evidence of both elevated anomaly intensity and mineralization-related geochemical patterns were categorized as no-priority samples. Spatial analysis of the prioritization results reveals that Priority 1 samples are predominantly concentrated within or in close proximity to well-documented mineralized zones and show a strong spatial correspondence with areas currently undergoing active exploration. In contrast, Priority 2 samples are more commonly distributed along the margins of these

zones or aligned with regional geological structures and drainage networks. Such patterns may reflect lateral extensions of mineralization, sediment transport effects, or concealed exploration targets characterized by lower anomaly intensity. No-priority samples largely represent the regional geochemical background; excluding them from subsequent exploration stages enables a more efficient allocation of exploration resources toward targets with higher geological confidence.

This study demonstrates that the targeted integration of fractal modeling and fuzzy FCM clustering leads to the generation of a decision-oriented, geologically consistent, and exploration-reliable prioritization map. While IFFAI plays a critical supporting role in enhancing spatial clarity and validating anomalous trends, the final prioritization scheme is fundamentally grounded in the coexistence of anomaly intensity and multielement mineralization signatures. This framework is readily transferable to other ore-forming elements and geological settings and can be effectively applied in the design of follow-up sampling programs, trenching site selection, and the prioritization of exploratory drilling as a cost-efficient and robust approach in early-stage mineral exploration.

Author contributions Hossein Mahdianfar: Conceptualization, methodology development, data processing and analysis, interpretation of results, original draft preparation, and manuscript writing. Mirmahdi Seyedrahimi-Niaraq: Data collection, assistance in data preprocessing, validation of results, and contribution to manuscript writing.

Funding This research did not receive any specific grant from funding agencies in the public, commercial, or not-for-profit sectors.

Data availability The datasets analyzed during the current study are not publicly available due to confidentiality agreements and data ownership restrictions imposed by the private mining company, but are available from the corresponding author on reasonable request.

Declarations

Competing interests The authors declare no competing interests.

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